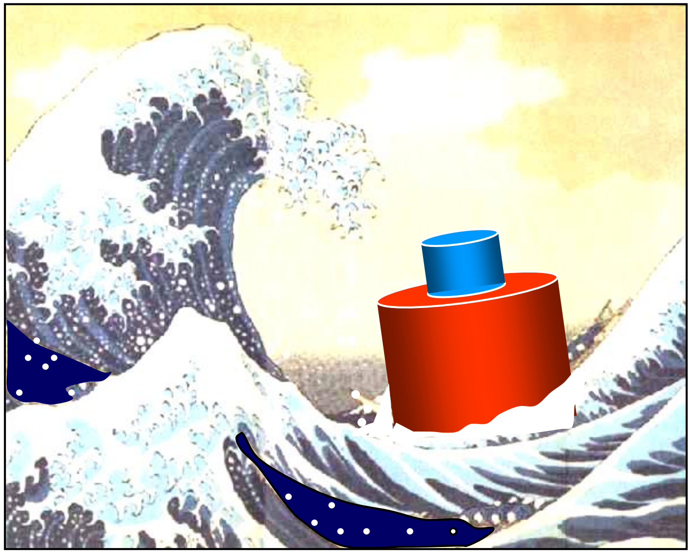
Università degli Studi di Firenze, 18-19 April 2012



WAVE ENERGY UTILIZATION

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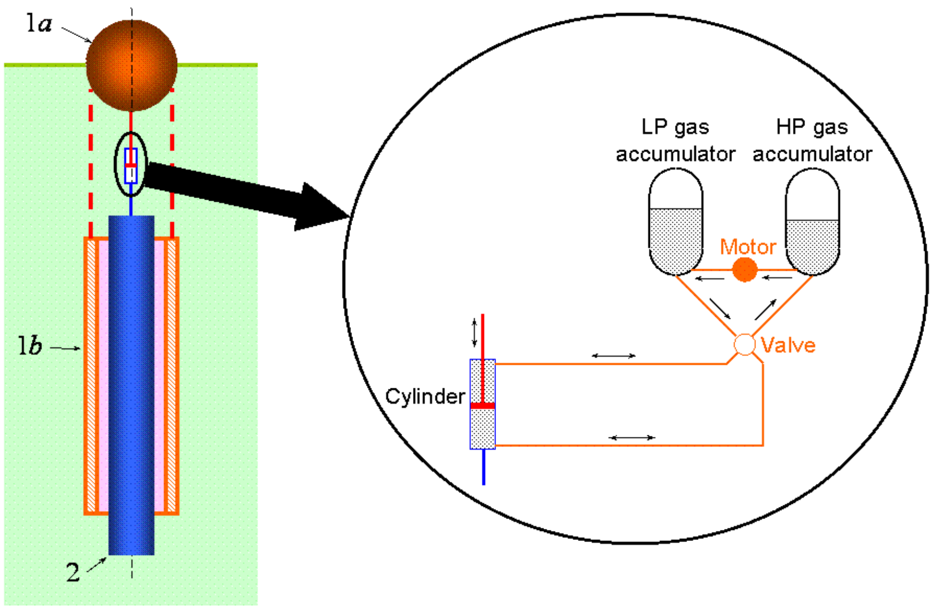
Part 3
Wave Energy Conversion Modelling

- **Introduction.**
- **Oscillating-body dynamics.**
- **Oscillating-Water-Column (OWC) dynamics.**
- **Model testing in wave tank.**



Introduction

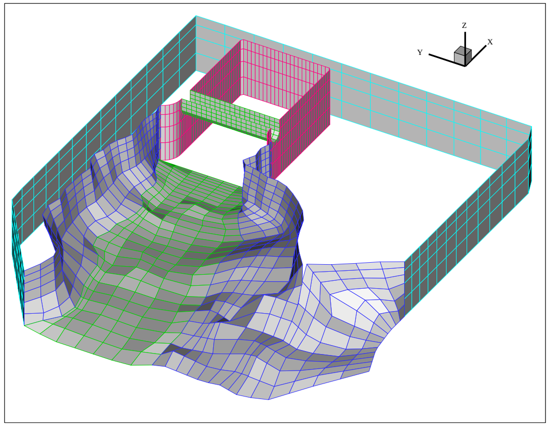
Steps in the development of a wave energy converter:

1. Basic conception of device

- **inventor(s)**
- **new patent or from previous concept**

2. Theoretical modelling (hydrodynamics, PTO, control,…)

- **evaluation (is device promising or not?)**
- **optimization, control studies, …**
- **requires high degree of specialization (universities, etc.)**



Introduction

3. Model testing in wave tank

- to complement and validate the theoretical/numerical modelling
- scales 1:100 (in small tanks) to 1:10 (in very large tanks)
- essential before full-sized testing in real sea

4. Technical demonstration: design, construction and testing of a large model (~1/4th scale) or full-sized prototype in real sea:

- the real proof of technical viability of the system
- cost: up to tens of M$

5. Commercial demonstration: several-MW plant in the open sea (normally a wave farm) with permanent connection to the electrical grid.

Introduction

Theoretical/numerical hydrodynamic modelling

- **Frequency-domain**
- **Time-domain**
- **Stochastic**

In all cases, <u>linear water wave theory</u> is assumed:
- **small amplitude waves and small body-motions**
- **real viscous fluid effects neglected**

<u>Non-linear water wave theory</u> may be used at a later stage to investigate some water flow details.

Introduction

Frequency domain model

Basic assumptions:

- **Monochromatic (sinusoidal) waves**
- **The system (input → output) is linear**

- **Historically the first model**
- **The starting point for the other models**

Advantages:

- **Easy to model and to run**
- **First step in optimization process**
- **Provides insight into device's behaviour**

Disadvantages:

- **Poor representation of real waves (may be overcome by superposition)**
- **Only a few WECs are approximately linear systems (OWC with Wells turbine)**

Introduction

Time-domain model

Basic assumptions:

- **In a given sea state, the waves are represented by a spectral distribution**

Advantages:

- **Fairly good representation of real waves**
- **Applicable to all systems (linear and non-linear)**
- **Yields time-series of variables**
- **Adequate for control studies**

Disadvantages:

- **Computationally demanding and slow to run**

Essential at an advanced stage of theoretical modelling

Introduction

Stochastic model

Basic assumptions:

- **In a given sea state, the waves are represented by a spectral distribution**
- **The waves are a Gaussian process**
- **The system is linear**

Advantages:

- **Fairly good representation of real waves**
- **Very fast to run in computer**
- **Yields directly probability density distributions**

Disadvantages:

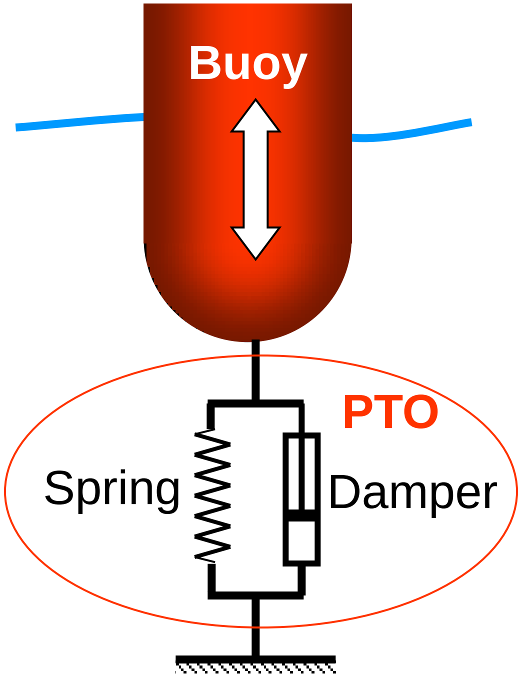
- **Restricted to approximately linear systems (e.g. OWCs with Wells turbines)**
- **Does not yield time-series of variables**

Oscillating-body dynamics

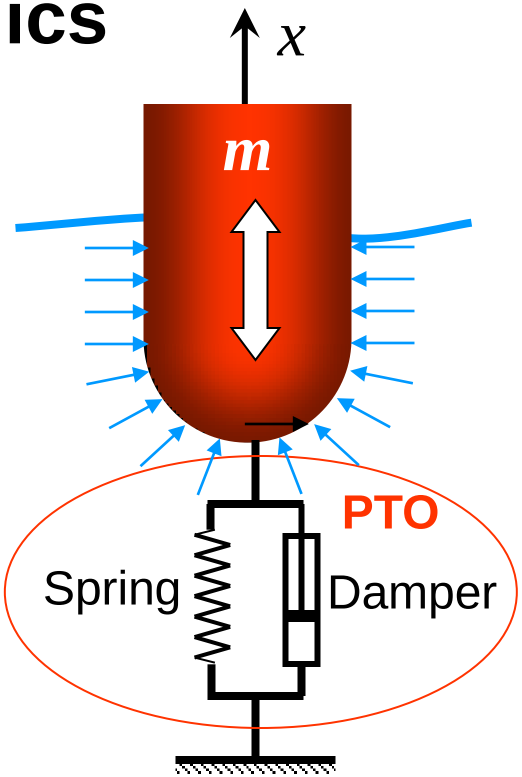
Most wave energy converters are complex (possibly multi-body) mechanical systems with several degrees of freedom.

We consider the simplest case:

- **A single floating body.**
- **One degree of freedom: oscillation in heave (vertical oscillation).**



Oscillating-body dynamics



Basic equation (Newton):

$$m\ddot{x} = f_h(t) + f_m(t)$$

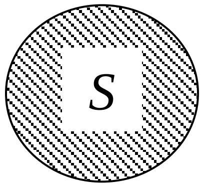
f_h: **on wetted surface**
f_m: **PTO**

$$f_h = \begin{cases} f_d = \\ f_r = \\ f_{hs} = -\rho g S x \end{cases}$$

f_d = **excitation force (incident wave)**

f_r = **radiation force (body motion)**

$f_{hs} = -\rho g S x$ = **hydrostatic restoring force (position)**



Cross-section

$$\boxed{m\ddot{x} = f_d + f_r - \rho g S x + f_m}$$

Oscillating-body dynamics

Frequency-domain analysis

- **Sinusoidal monochromatic waves, frequency** ω
- **Linear system**

Oscillating-body dynamics

$$\boxed{m\ddot{x} = f_d + f_r - \rho g S x + f_m}$$

$$\boxed{f_r = -A\ddot{x} - B\dot{x}}$$

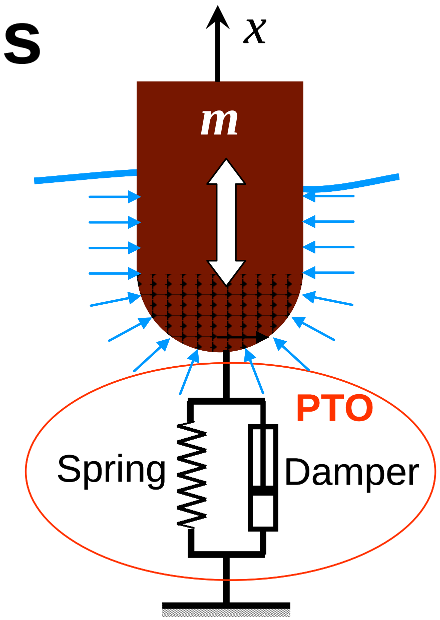
B = radiation damping

A = added mass

$$\boxed{f_m = -Kx - C\dot{x}}$$

Linear spring

Linear damper



***A* and *B* to be computed (commercial codes WAMIT, AQUADYN, ...) for given *ω* and body geometry.**

$$\boxed{(m + A)\ddot{x} + (B + C)\dot{x} + (\rho g S + K)x = f_d}$$

mass

added mass

radiation damping

PTO damping

buoyancy

PTO spring

Excitation force

Oscillating-body dynamics

$$\boxed{(m + A)\ddot{x} + (B + C)\dot{x} + (\rho g S + K)x = f_d}$$

Method of solution: $(e^{i\omega t} = \cos \omega t + i \sin \omega t)$

- **Regular waves**
- **Linear system**

$\Rightarrow$ $x(t) = \mathrm{Re}\left\{X_0 e^{i\omega t}\right\}, \quad f_d = \mathrm{Re}\left\{F_d e^{i\omega t}\right\}$

or simply $x(t) = X_0 e^{i\omega t}, \quad f_d = F_d e^{i\omega t}$

Note : X_0, F_d are in general complex amplitudes

$$\frac{|F_d|}{\text{wave amplitude}} = \Gamma(\omega) \Rightarrow \text{to be computed for given } \omega \text{ and body geometry}$$

$$\boxed{X_0 = \frac{F_d}{-\omega^2(m + A) + i\omega(B + C) + \rho g S + K}}$$

Oscillating-body dynamics

$$\boxed{X_0 = \frac{F_d}{-\omega^2(m+A) + i\omega(B+C) + \rho gS + K}}$$

Power = force × velocity

Time-averaged power absorbed from the waves :

$$\overline{P} = \frac{1}{8B}\left|F_d\right|^2 - \frac{B}{2}\left|i\omega X_0 - \frac{F_d}{2B}\right|^2$$

= 0

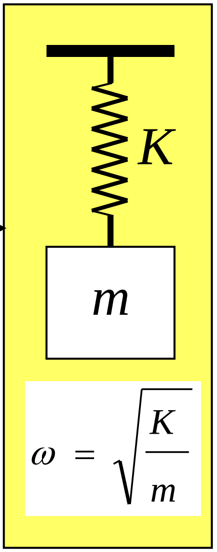
Note: for given body and given wave amplitude and frequency ω, B and F_d are fixed.

Then, the absorbed power $\overline{P}$ will be maximum when :

$$i\omega X_0 = \frac{F_d}{2B} \quad \begin{cases} \boxed{\omega = \sqrt{\dfrac{\rho gS + K}{m+A}}} \\ \boxed{B = C} \end{cases}$$

Resonance condition

Radiation damping = PTO damping



Oscillating-body dynamics

Capture width L : measures the power absorbing capability of device (like power coefficient of wind turbines)

$$L = \frac{\overline{P}}{E} \quad \begin{cases} \overline{P} = \text{absorbed power} \\ E = \text{energy flux of incident wave per unit crest length} \end{cases}$$

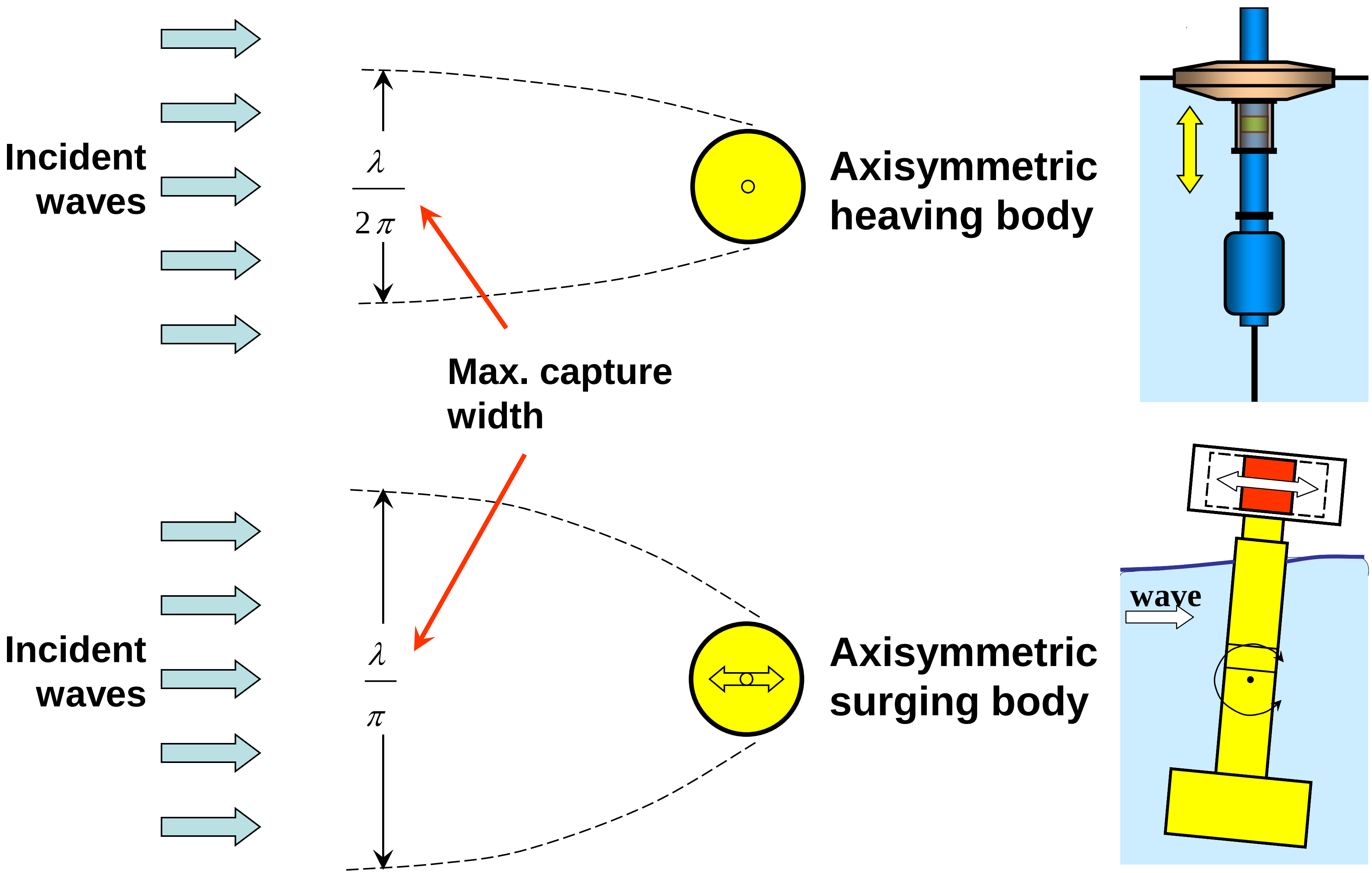
For an axisymmetric body oscillating in heave (vertical oscillations), it can be shown (1976) that

$$\boxed{\overline{P}_{\max} = \frac{E\lambda}{2\pi}} \quad \text{or} \quad \boxed{L_{\max} = \frac{\lambda}{2\pi}}$$

Note: $L_{\max}$ may be larger than width of body

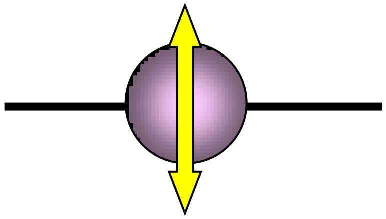
For wind turbines, Betz's limit is $C_P = 0.593$

Oscillating-body dynamics



Oscillating-body dynamics

Example: hemi-spherical heaving buoy of radius a



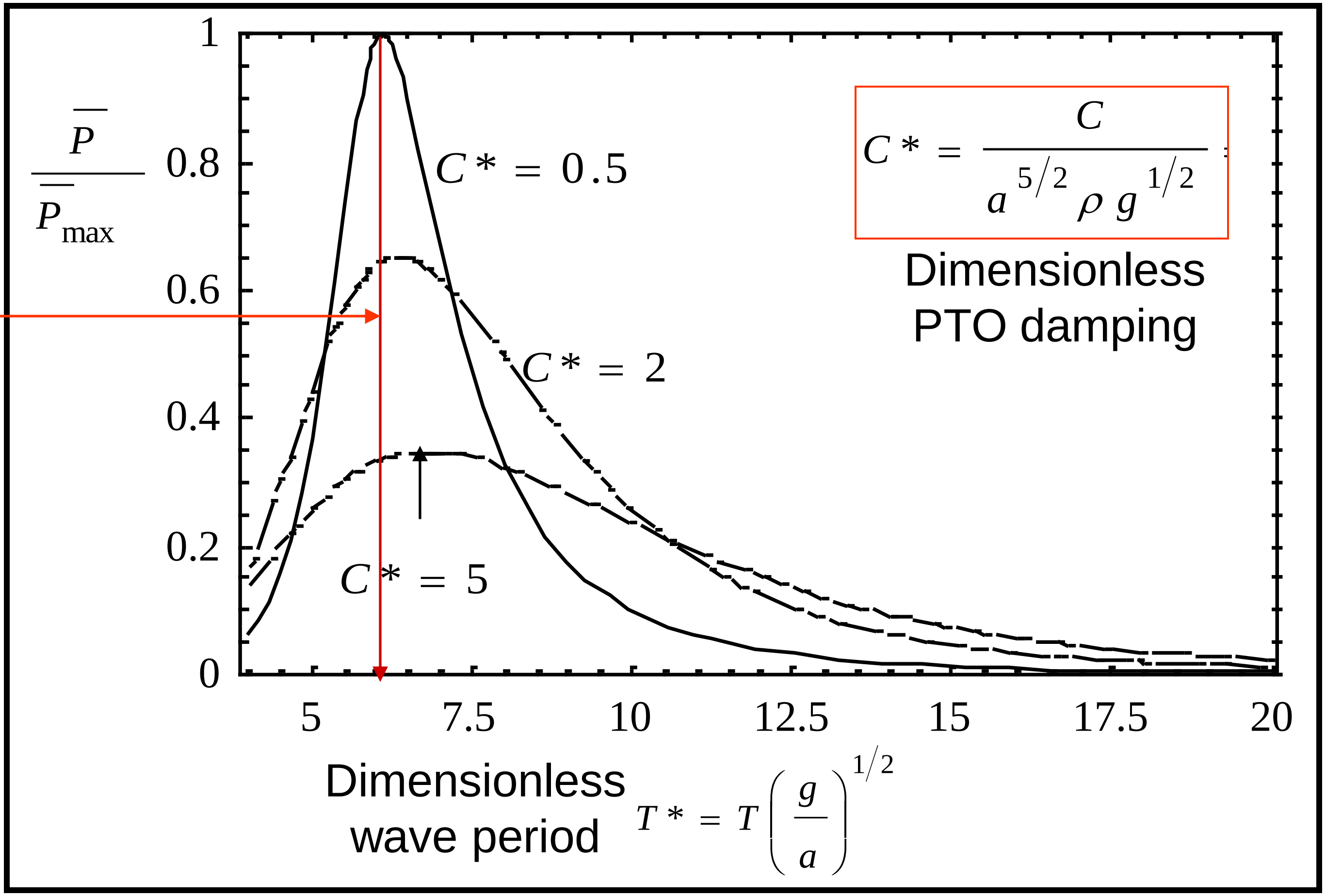
No spring, $K = 0$

$\overline{P} = \overline{P}_{max}$ for

$$T^* = T\left(\frac{g}{a}\right)^{1/2} = 6$$

If $T = 9$ s

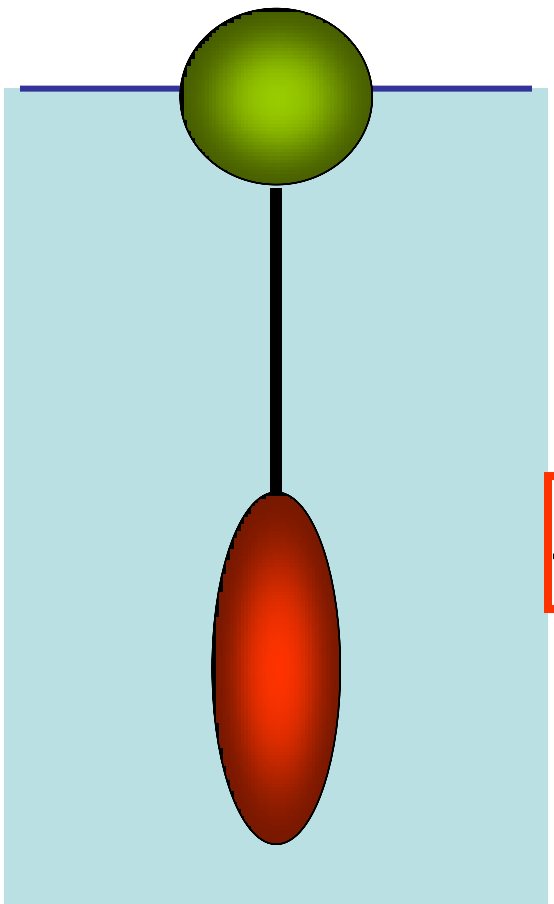
$$a_{opt} = 22 \text{ m}$$



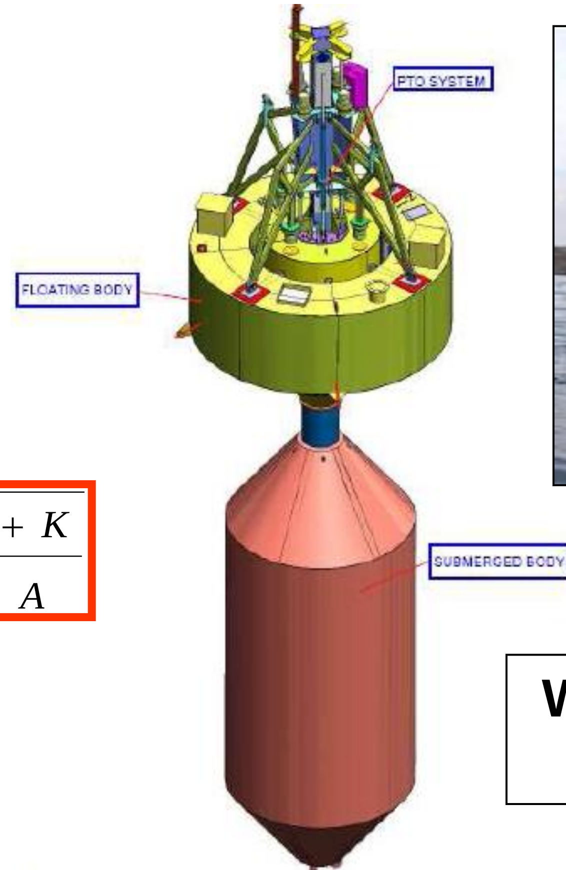
Oscillating-body dynamics

How to increase the resonance frequency of a given "small" floater?

Attach a deeply submerged body.



$$\boxed{\omega = \sqrt{\frac{\rho g S + K}{m + A}}}$$



Wavebob, Ireland

Oscillating-body dynamics

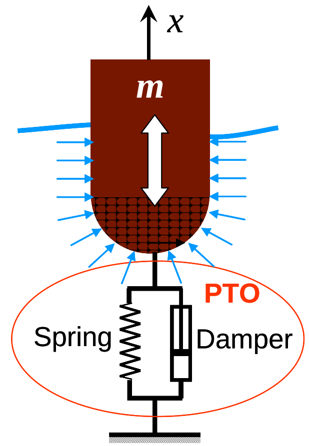
Time-domain analysis

- **Regular or irregular waves**
- **Linear or non-linear PTO**

- May be require significant time-computing
- Yields time-series
- Essential for control studies

- **A.F. de O. Falcão, "Modelling and control of oscillating-body wave energy converters with hydraulic power take-off and gas accumulator",** ***Ocean Engineering*****, vol. 34, pp. 2021-2032, 2007.**
- **A.F. de O. Falcão, "Phase control through load control of oscillating-body wave energy converters with hydraulic PTO system",** ***Ocean Engineering*****, vol. 35, pp. 358-366, 2008.**

Oscillating-body dynamics
Time domain



From Fourier transform techniques:

$$(m + A_\infty)\,\ddot{x}(t) = f_d(t) - \rho g S\, x(t) - \int_{-\infty}^{t} L(t-\tau)\,\ddot{x}(\tau)\,d\tau + f_m(x, \dot{x}, t) \quad (1)$$

added mass **excitation** **hydrostatic** **radiation** **PTO**

forces

$$L(t) = \frac{1}{2\pi}\int_0^\infty \frac{B(\omega)}{\omega}\sin \omega t \, d\omega$$ **memory function**

$$f_d(t) = \sum_n f_{d,n}(t)$$ **from $\Gamma(\omega)$ and spectral distribution (Pierson-Moskowitz, …)**

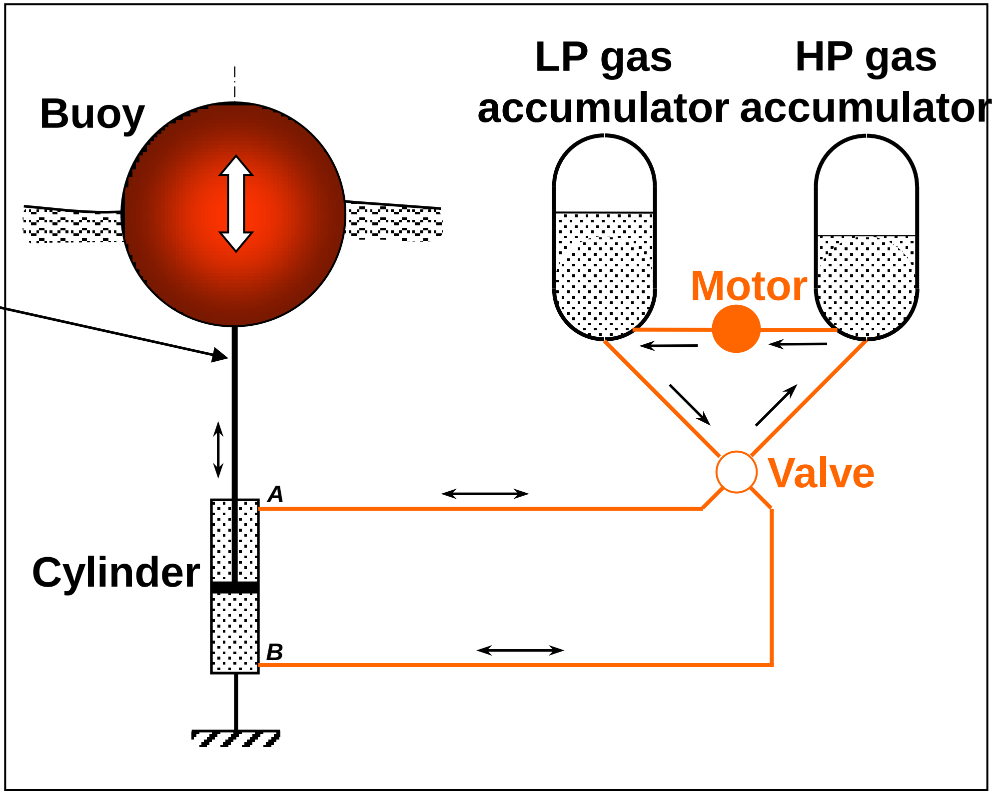
Equation (1) to be numerically integrated

Oscillating-body dynamics

Example: Heaving buoy with hydraulic PTO (oil)

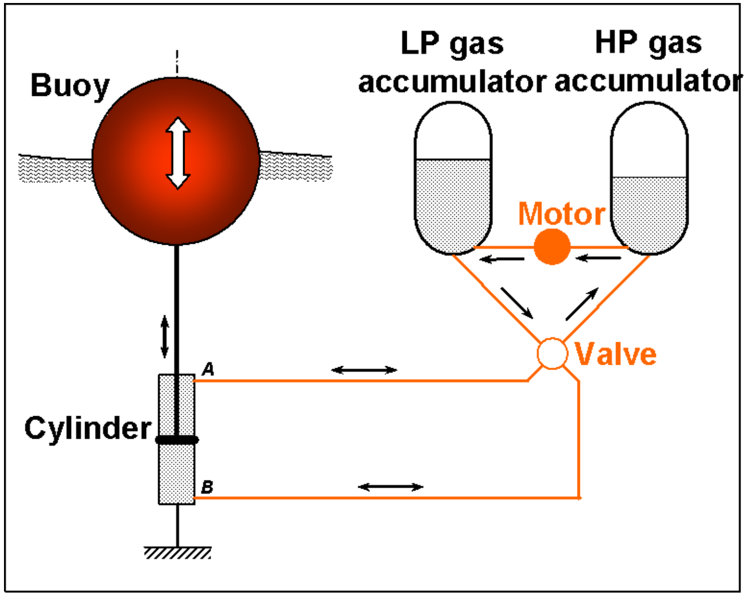
- Hydraulic cylinder (ram)
- HP and LP gas accumulator
- Hydraulic motor

PTO force:
Coulomb type (imposed by pressure in accumulator, piston area and rectifying valve system)

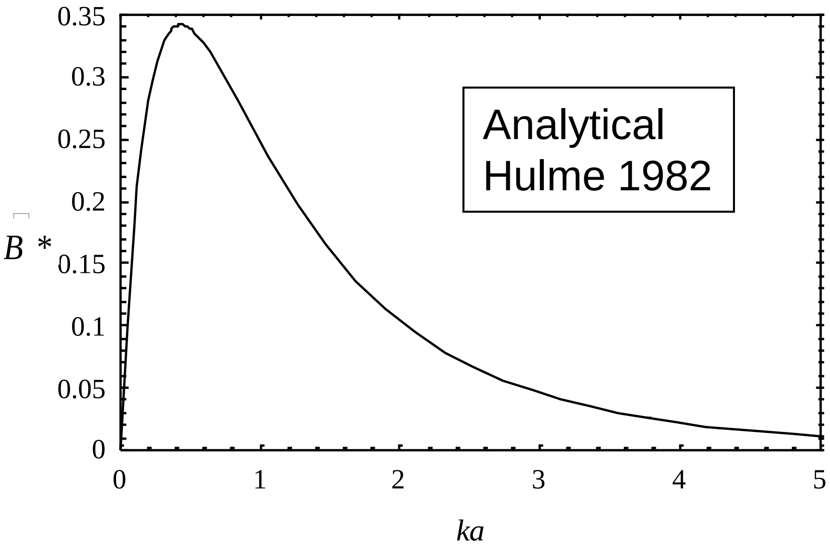


Oscillating-body dynamics

Example:
- **Hemispherical buoy, radius** $= a$

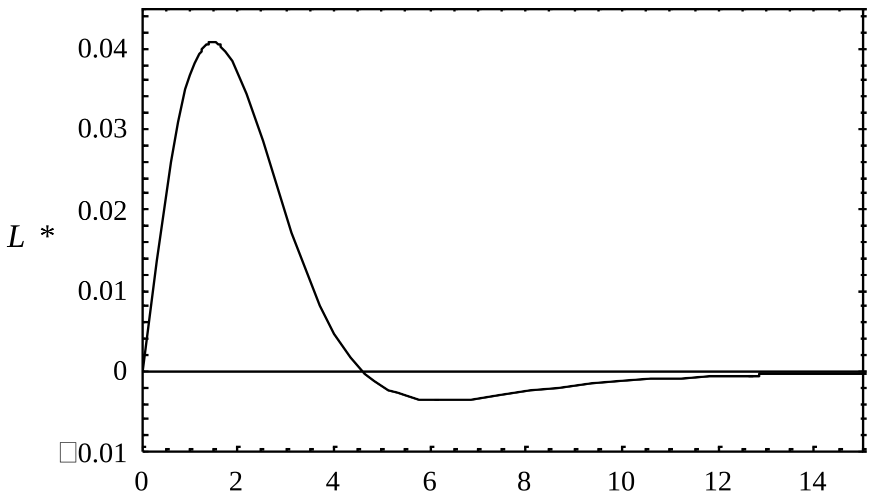


Dimensionless radiation damping coefficient



Dimensionless radius

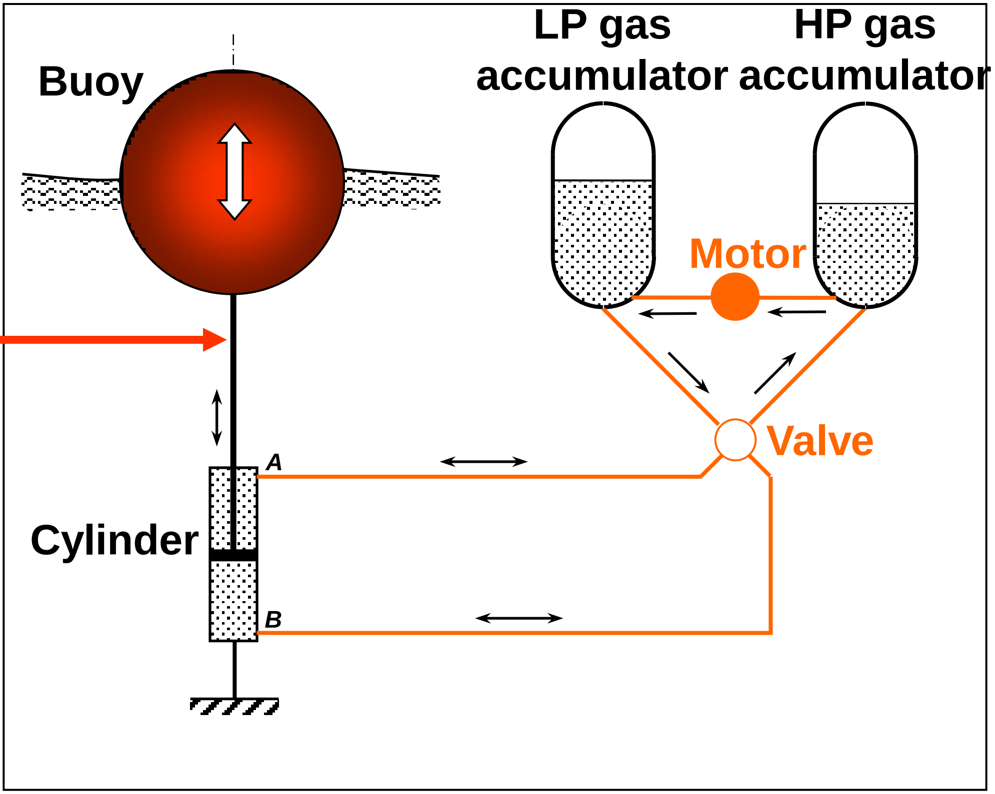
Dimensionless memory function



Dimensionless time $t^* = t\sqrt{\dfrac{g}{a}}$

Oscillating-body dynamics

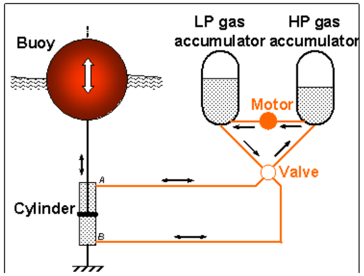
External PTO force:
Coulomb type (imposed by pressure in accumulator, piston area and rectifying valve system)



Irregular waves with H_s, T_e and Pierson-Moskowitz spectral distribution

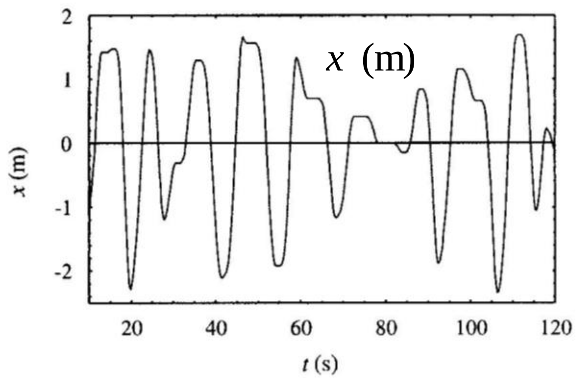
$$S_{\zeta}(\omega) = 263\ H_s^2 T_e^{-4} \omega^{-5} \exp(-1054\ T_e^{-4} \omega^{-4})$$

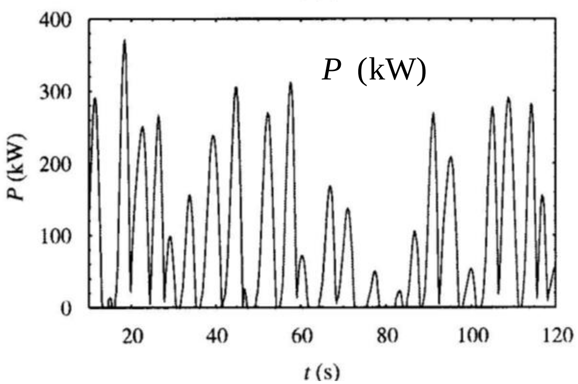
Oscillating-body dynamics



Sphere radius $a = 5$ m

Sea state $H_s = 3$ m, $T_e = 11$ s

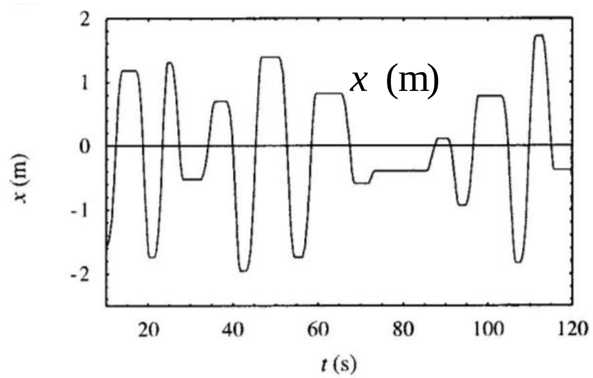


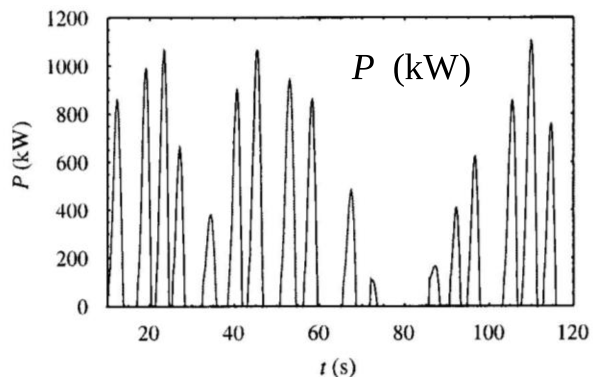


Under damped

External force 200 kN

$\overline{P} = 83.1$ kW

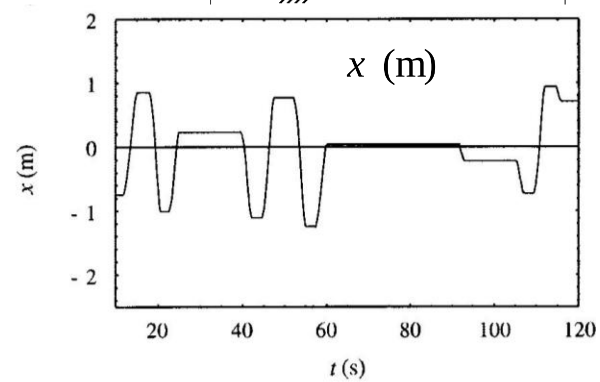


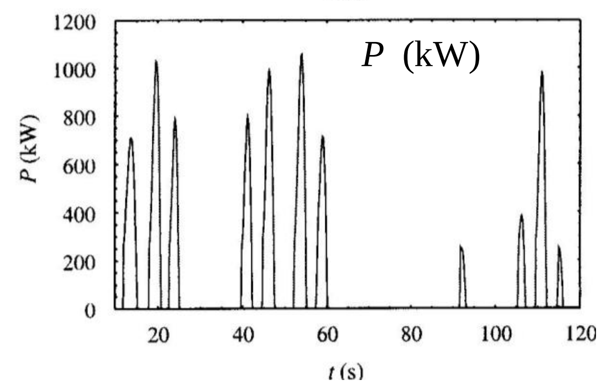


Optimally damped

External force 647 kN

$\overline{P} = 178.4$ kW





Over damped

External force 1000 kN

$\overline{P} = 97.0$ kW

Oscillating-body dynamics

For **point absorbers** (relatively small bodies) the resonance frequency of the body is in general much larger than the typical wave frequency of sea waves:

- No resonance can be achieved.
- Poor energy absorption.

How to increase energy absorption?

Phase control !

Oscillating-body dynamics

Phase-control by **latching**

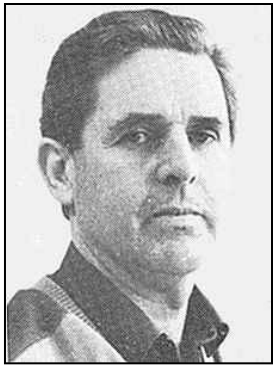
Whenever the body velocity comes down to zero, keep the body fixed for an appropriate perid of time.

This is an artificial way of reducing the frequency of the body free-oscillations, and achieving resonance.

Phase-control by latching was introduced by Falnes and Budal

J. Falnes, K. Budal, Wave-power conversion by power absorbers. *Norwegian Maritime Research*, 6, 2-11, 1978.

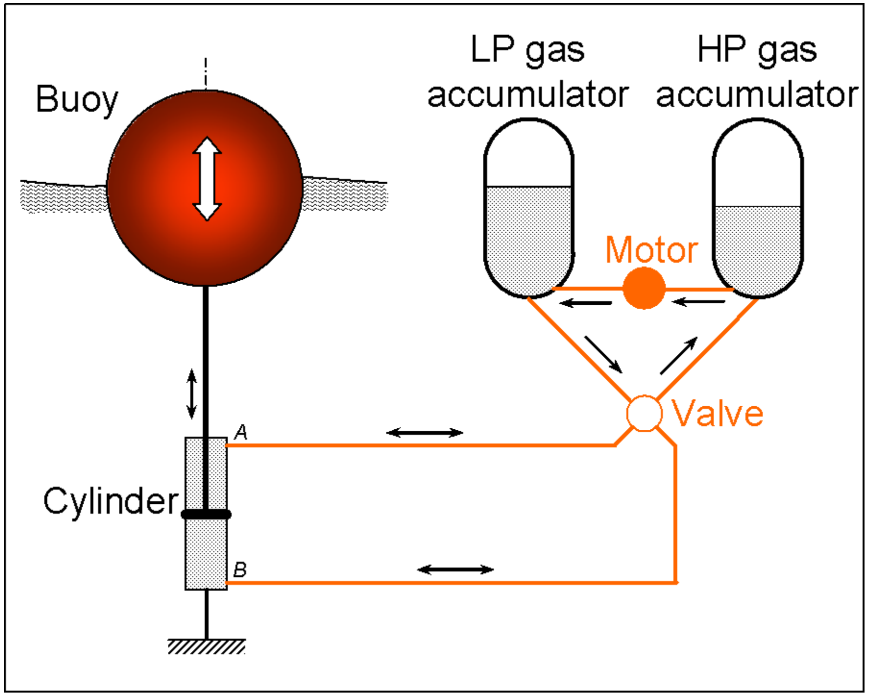
Johannes Falnes



Kjell Budall (1933-89)

Oscillating-body dynamics

How to achieve **phase-control by latching** in a floating body with a hydraulic power-take-off mechanism?



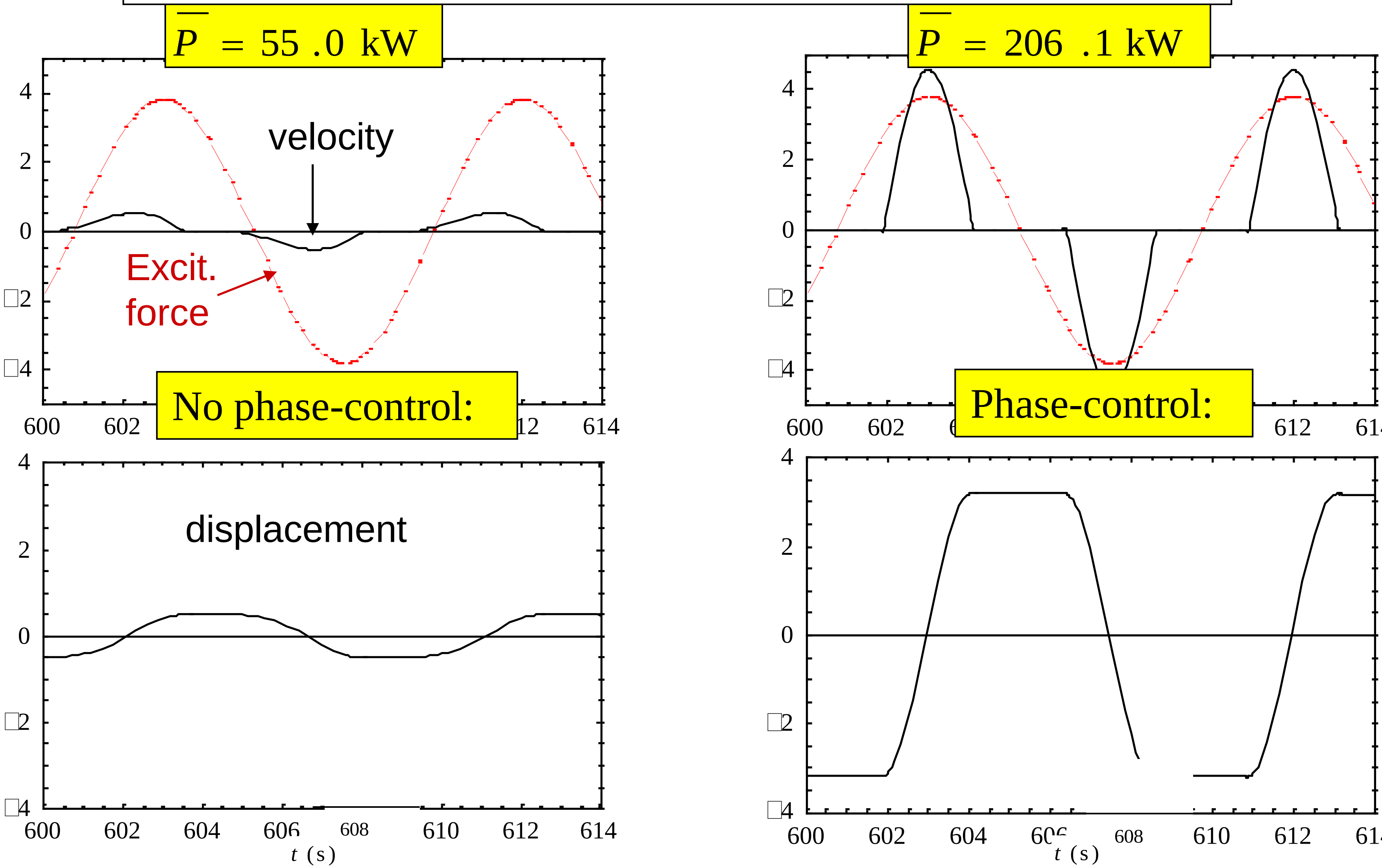
Introduce a delay in the release of the latched body.

How?

Increase the resisting force the hydrodynamic forces have to overcome to restart the body motion.

Oscillating-body dynamics

Regular waves: *T* = 9 s, amplitude 0.67 m

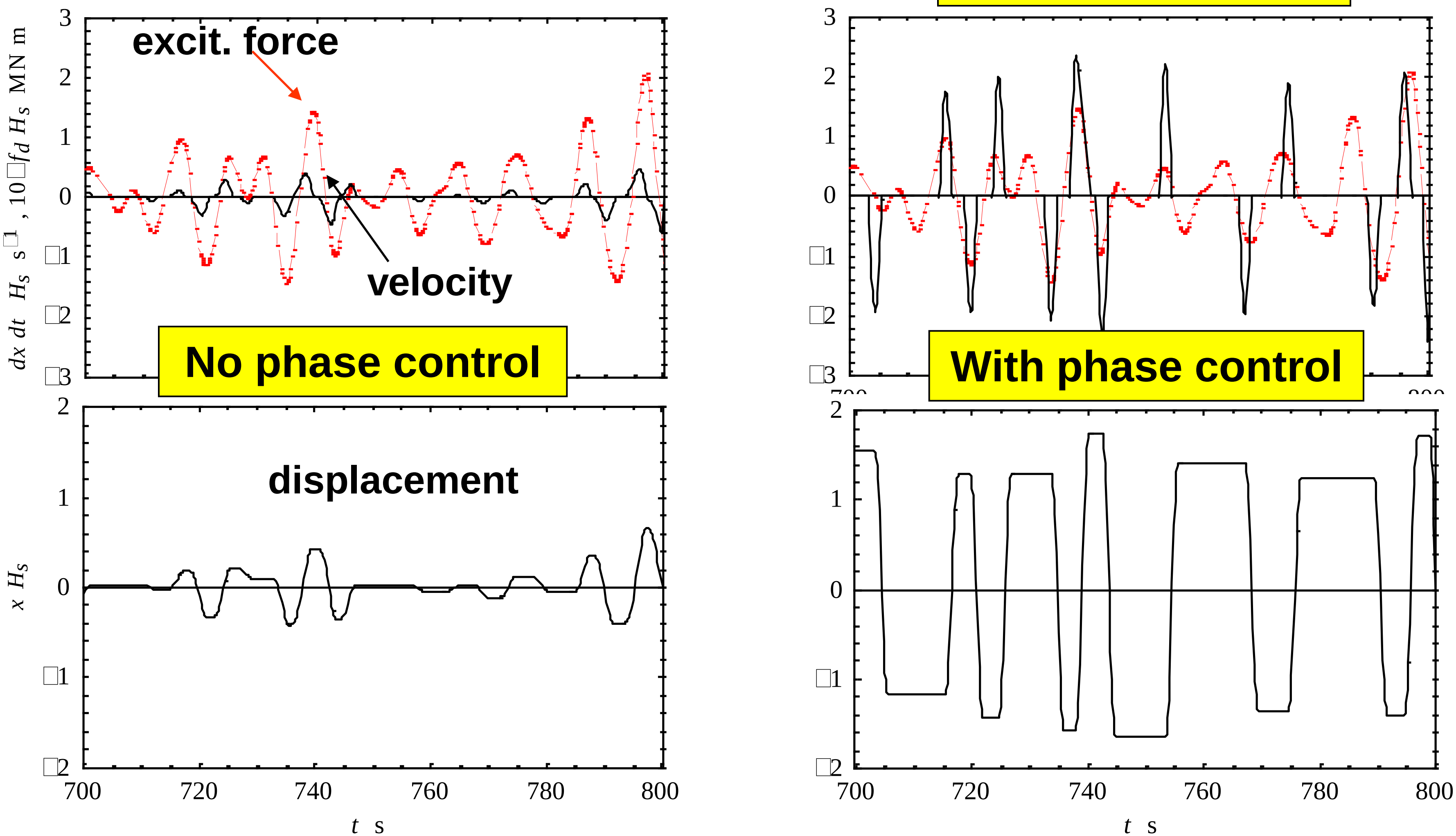


Oscillating-body dynamics

Irregular waves: T_e = 9 s, H_s = 2 m

$\overline{P} / H_s^2 = 41.2 \text{ kW/m}^2$

$\overline{P} / H_s^2 = 114 \text{ kW/m}^2$



Oscillating-body dynamics

Phase by latching may significantly increase the amount of absobed energy by point absorbers.

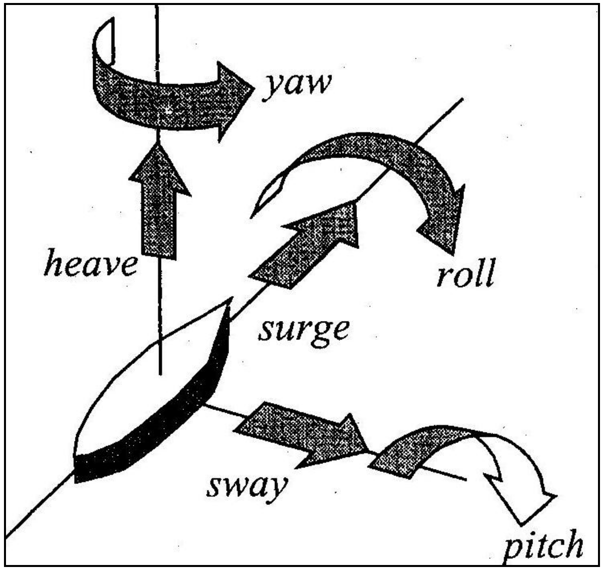
Problems with latching phase control:

- Latching forces may be very large.
- Latching control is less effective in two-body WECs.

Apart from latching, there are forms of phase control (reactive, unlatching, …).

Phase control is being investigated by several teams as a way of enhancing device performance.

Oscillating-body dynamics



Several degrees of freedom

- Each body has 6 degrees of freedom
- A WEC may consist of *n* bodies (*n* >1)

All these modes of oscillation interact with each other through the wave fields they generate.

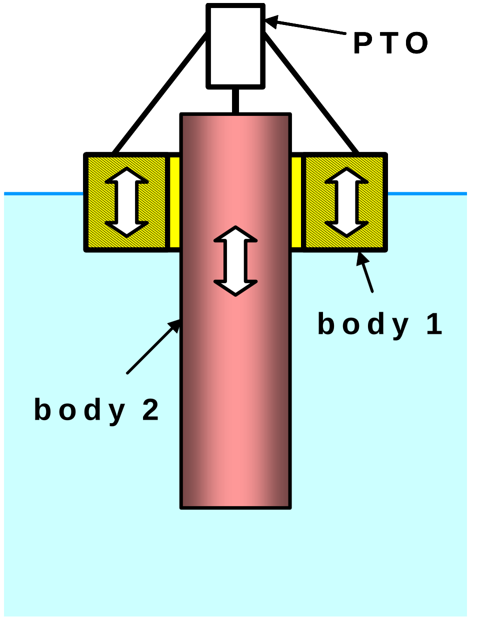
Number of dynamic equations = 6*n*

The interference between modes affects:

- added masses
- radiation damping coefficients

Hydrodynamic coefficients A_{ij}, B_{ij} are defined accordingly.

They can be computed with commercial software (WAMIT, …).

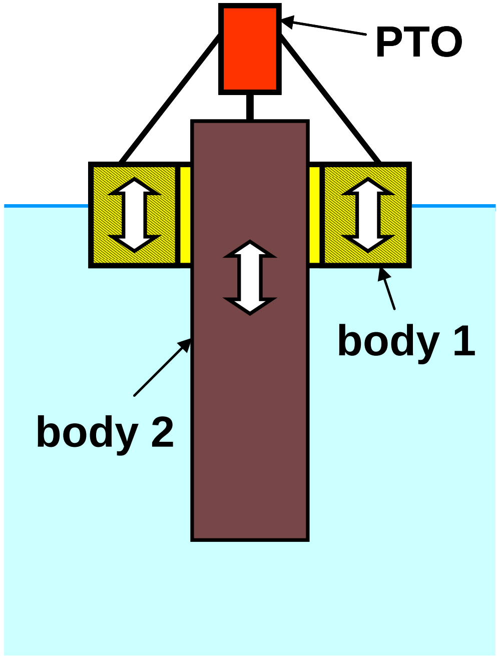


Oscillating-body dynamics

Several degrees of freedom

Example: heaving bodies 1 and 2 reacting against each other.

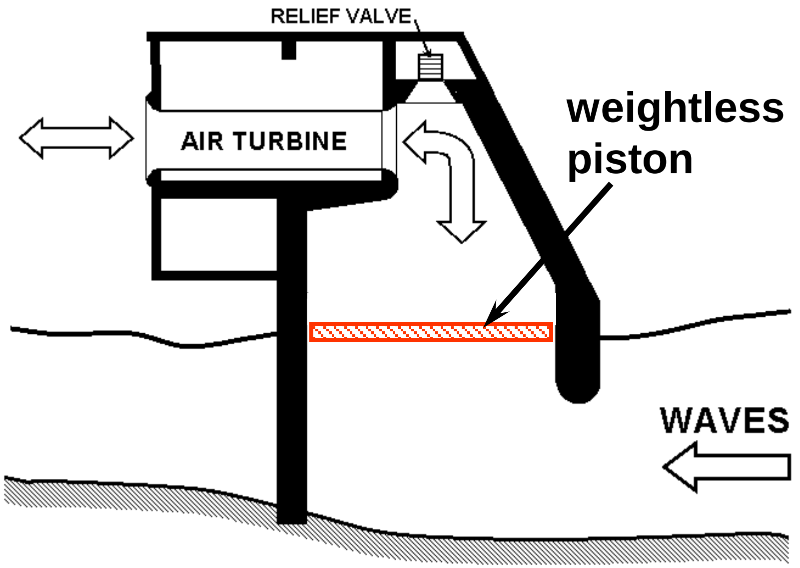
$$\begin{cases} (m_1 + A_1)\ddot{x}_1 + B_1\dot{x}_1 + \rho g S_1 x_1 + C(\dot{x}_1 - \dot{x}_2) + K(x_1 - x_2) \\ \quad + A_{12}\ddot{x}_2 + B_{12}\dot{x}_2 = f_{d1} \\ (m_2 + A_2)\ddot{x}_2 + B_2\dot{x}_2 + \rho g S_2 x_2 - C(\dot{x}_1 - \dot{x}_2) - K(x_1 - x_2) \\ \quad + A_{12}\ddot{x}_1 + B_{12}\dot{x}_1 = f_{d2} \end{cases}$$



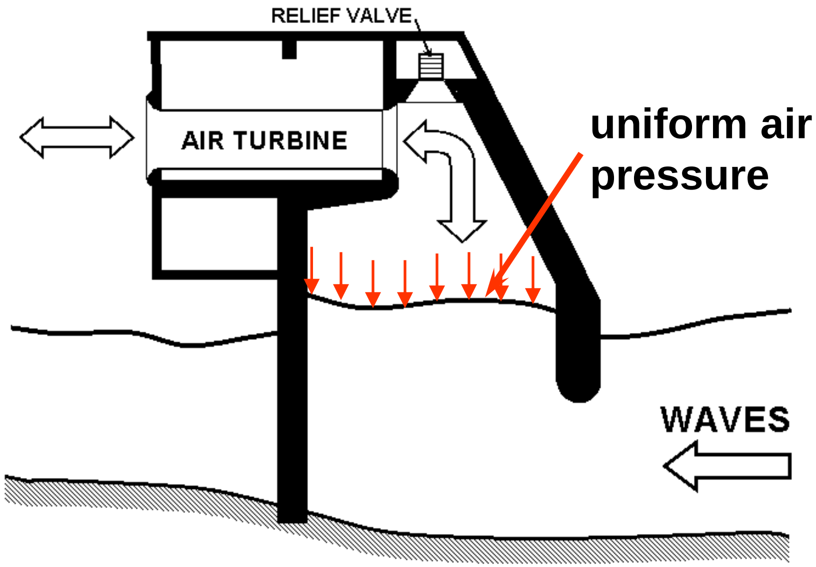
Note: $A_{12} = A_{21}, \quad B_{12} = B_{21}$

OWC Dynamics

Two different approaches to modelling:

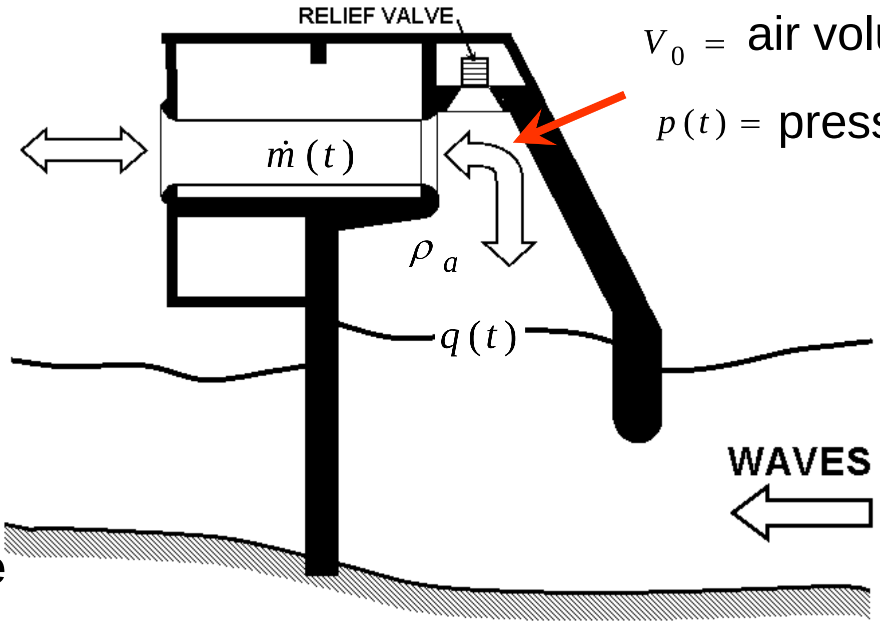


Oscillating body (piston) model
(rigid free surface)



Uniform pressure model
(deformable free surface)

OWC Dynamics



$V_0 =$ air volume

$p(t) =$ pressure

$q(t) =$ **volume-flow rate displace by free-surface**

$\dot{m}(t) =$ **mass-flow rate of air through turbine**

$\rho_a =$ **air density** $p(t) =$ **air pressure**

Conservation of air mass (linearized)

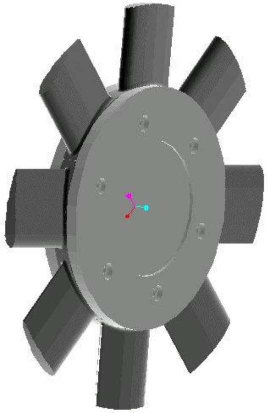
$$\frac{\dot{m}(t)}{\rho_a} = q(t) - \frac{V_0}{\rho_a c_a^2} \frac{dp(t)}{dt}$$

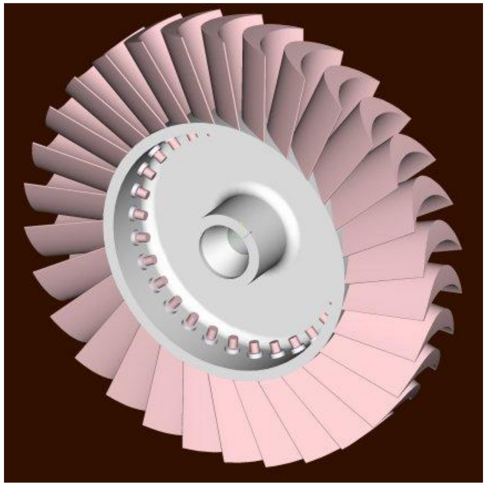
$$\text{flow rate} \quad q = \begin{cases} q_{exc} & \text{excitation} \\ q_r & \text{radiation} \end{cases}$$

Effect of air compressibility

OWC Dynamics

Air turbine
$$\begin{cases} N = \text{rotational speed} \\ D = \text{rotor diameter} \\ P_t = \text{power output} \\ p = \text{pressure head} \end{cases}$$





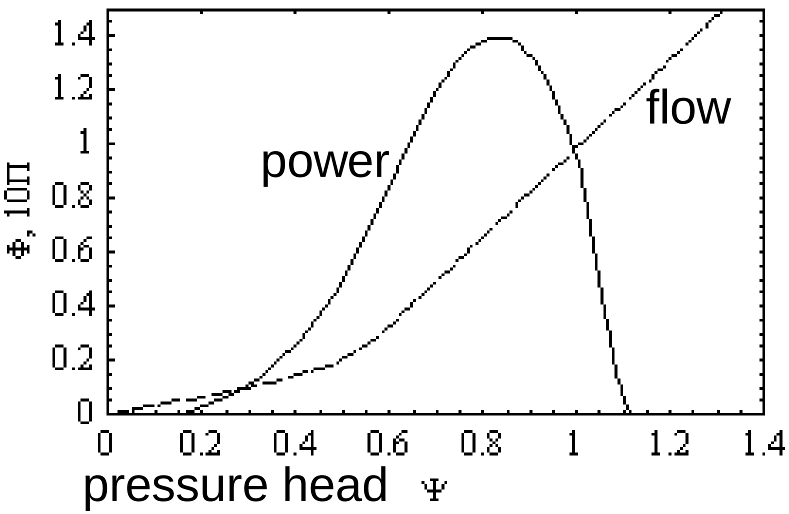
In dimensionless form:

$$\Phi = \frac{\dot{m}}{\rho_a N D^3} \qquad \Psi = \frac{p}{\rho_a N^2 D^2} \qquad \Pi = \frac{P_t}{\rho_a N^3 D^5}$$

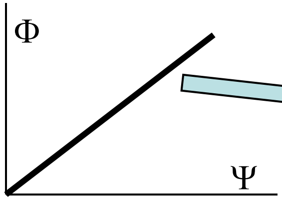
flow   pressure head   power

Performance curves of turbine (dimensionless form):

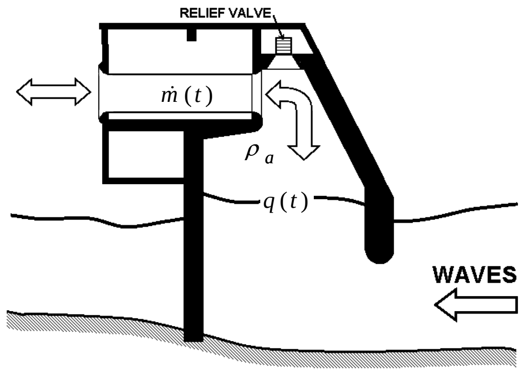
$$\Phi = f_w(\Psi), \qquad \Pi = f_P(\Psi)$$



OWC Dynamics



Frequency domain



Linear air turbine $\Phi = K\Psi$

$$\{p(t), \dot{m}, q(t), q_r(t), q_{\text{exc}}(t)\} = \{P, \dot{M}, Q, Q_r, Q_{\text{exc}}\} e^{i\omega t}$$

$$\frac{Q_r(\omega)}{P(\omega)} = -B(\omega) - iC(\omega) \quad \begin{cases} B = \text{radiation conductance} \\ C = \text{radiation susceptance} \end{cases}$$

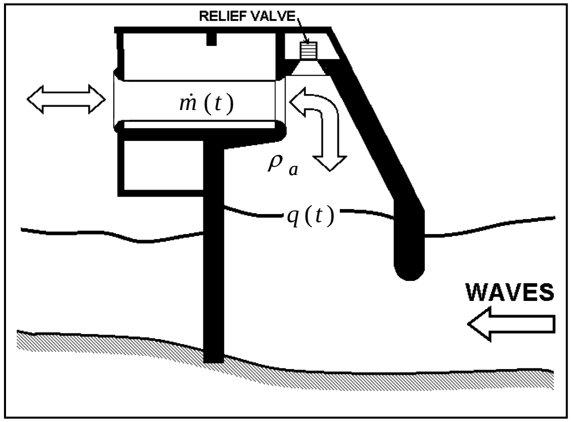
$$\left| Q_{\text{exc}}(\omega) \right| = \Gamma(\omega) A_w$$

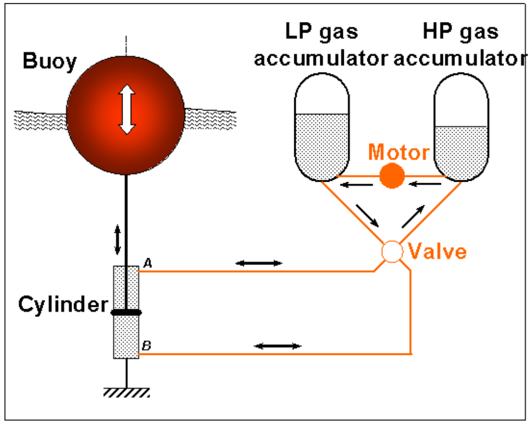
excitation coeff.

wave ampl.

$$P = \frac{Q_{\text{exc}}}{\dfrac{KD}{\rho_a N} + B + i\left(C + \dfrac{\omega V_0}{\rho_a c_a^2} \right)}$$

OWC Dynamics



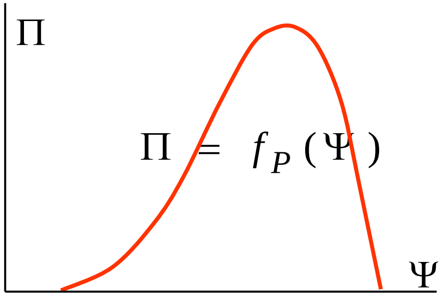


$$P = \frac{Q_{\text{exc}}}{\dfrac{KD}{\rho_a N} + B + i\left(C + \dfrac{\omega V_0}{\rho_a c_a^2}\right)}$$

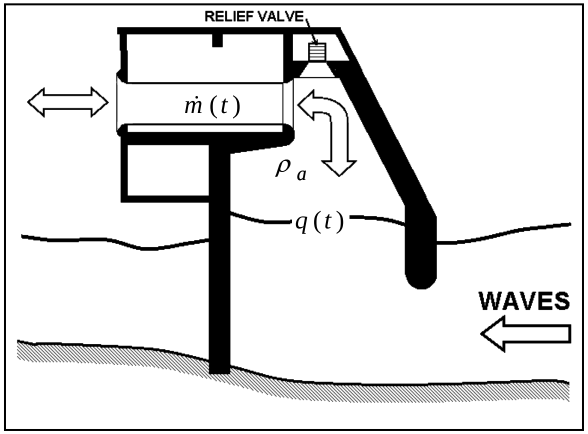
$$X_0 = \frac{F_d}{-\omega^2 (m + A) + i\omega (B + C) + \rho g S + K}$$

air pressure $p(t) = \mathrm{Re}\left(P e^{i\omega t}\right)$

power output : $\boxed{\Pi(t) = \dfrac{P_t(t)}{\rho_a N^3 D^5} = f_P\left(\underbrace{\dfrac{p(t)}{\rho_a N^2 D^2}}_{\Psi}\right)}$



OWC Dynamics



Time domain:

- **Linear or non-linear turbine**

$$\boxed{\frac{V_0}{\rho_a c_a^2}\frac{dp(t)}{dt}+\frac{\dot{m}(t)}{\rho_a}-\int_{-\infty}^{t} g_r(t-\tau)\,p(\tau)\,d\tau = q_i(t)}$$

$$\text{turbine flow vs pressure curve} \quad \dot{m} = \rho_a N D^3 f_w\left(\frac{p}{\rho_a N^2 D^2}\right)$$

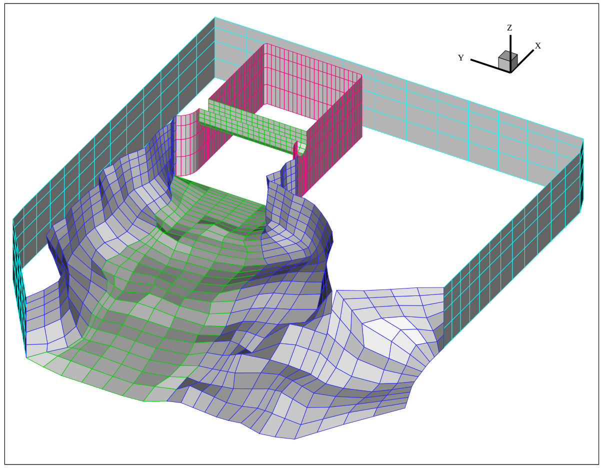
$$\text{memory function} \quad g_r(t) = -\frac{2}{\pi}\int_0^{\infty} B(\omega)\cos\omega t\, d\omega$$

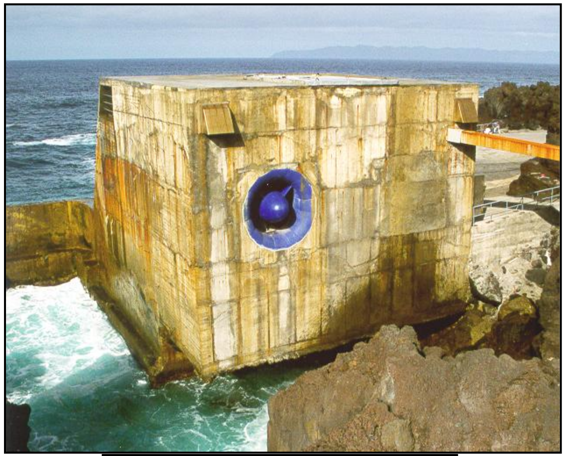
To be integrated numerically for *p(t)*

$$\text{power output} : \Pi(t) = \frac{P_t(t)}{\rho_a N^3 D^5} = f_P\left(\frac{p(t)}{\rho_a N^2 D^2}\right)$$

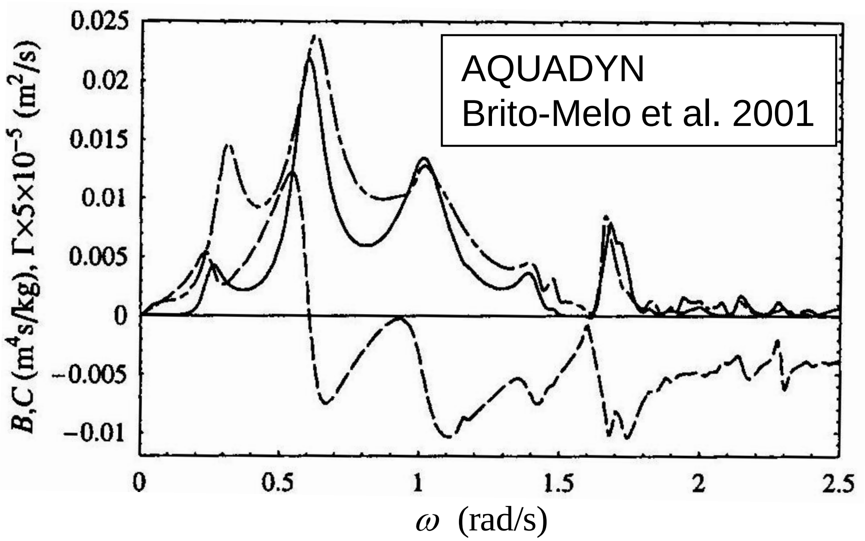
OWC Dynamics

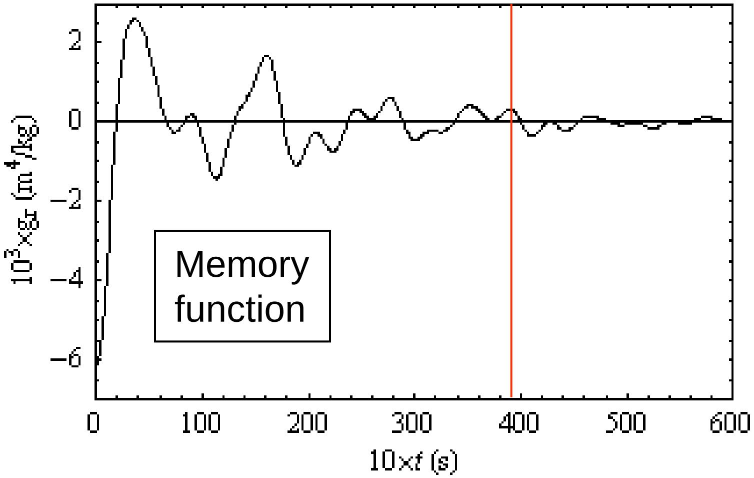
Numerical application





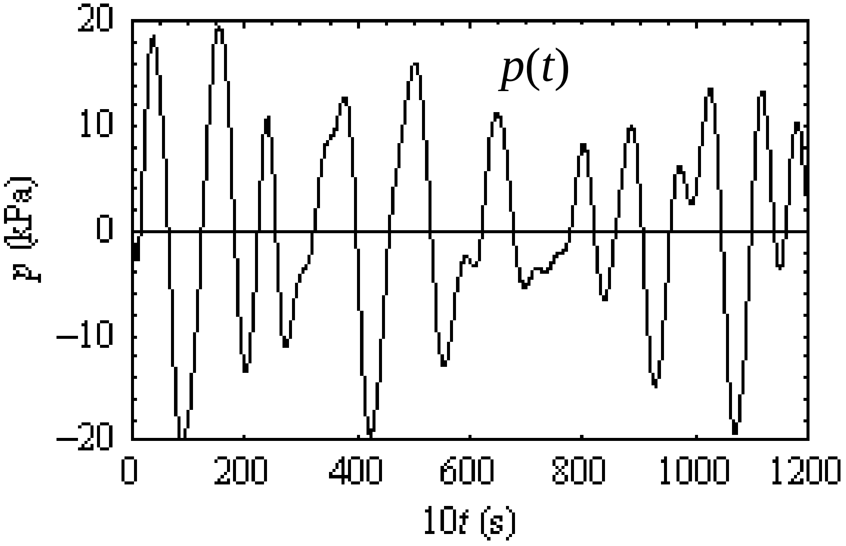
Pico OWC



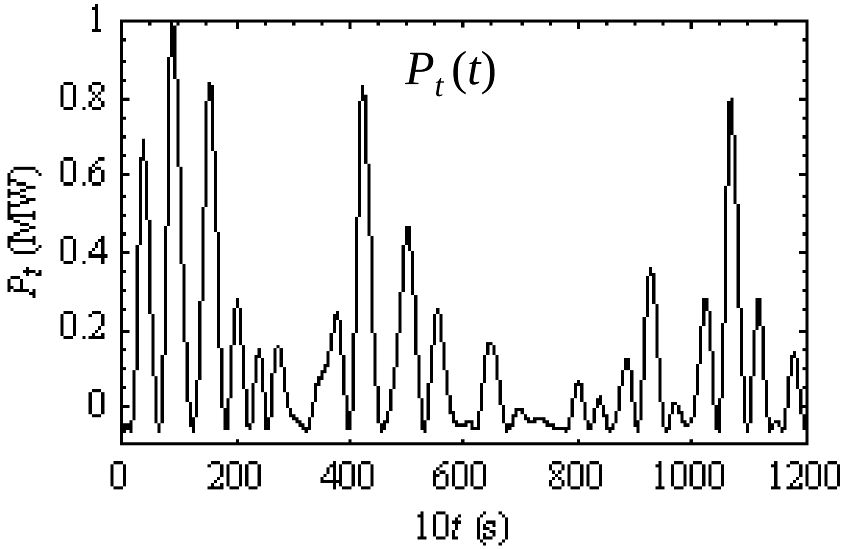


OWC Dynamics

Numerical application



Air pressure in chamber



Power

Results from time-domain modelling of <u>impulse turbine</u> over $\Delta t = 120$ s

- Turbine $D = 1.5$ m, $N = 115$ rad/s (1100 rpm)
- Sea state $H_s = 3$ m, $T_e = 11$ s
- Average power output from turbine 97.2 kW

OWC Dynamics

Stochastic modelling

- **Irregular waves**
- **Linear air-turbine**

- Much less time-consuming than time-domain analysis
- Appropriate for optimization studies

- **A.F. de O. Falcão, R.J.A. Rodrigues, "Stochastic modelling of OWC wave power performance", *Applied Ocean Research*, Vol. 24, pp. 59-71, 2002.**
- **A.F. de O. Falcão, "Control of an oscillating water column wave power plant for maximum energy production", *Applied Ocean Research*, Vol. 24, pp. 73-82, 2002.**
- **A.F. de O. Falcão, "Stochastic modelling in wave power-equipment optimization: maximum energy production versus maximum profit". *Ocean Engineering*, Vol. 31, pp. 1407-1421, 2004.**

OWC Dynamics

Stochastic modelling

Wave climate represented by a set of sea states

- **For each sea state: H_s, T_e, freq. of occurrence ϕ.**
- **Incident wave is random, Gaussian, with known frequency spectrum.**

WAVES → **OWC** → **AIR PRESSURE** → **TURBINE**

WAVES: Random, Gaussian

OWC: Linear system. Known hydrodynamic coefficients

AIR PRESSURE: Random, Gaussian rms: σ_p

TURBINE: Known performance curves

TURBINE ↓ **TURBINE SHAFT POWER**

TURBINE SHAFT POWER → **GENERATOR** → **ELECTRICAL POWER OUTPUT**

TURBINE SHAFT POWER: Time-averaged

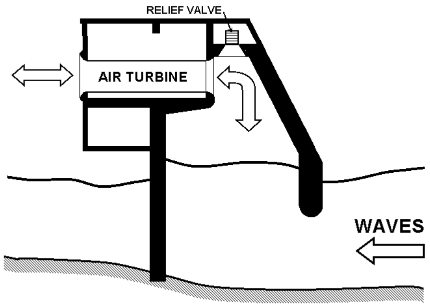
GENERATOR: Electrical efficiency

ELECTRICAL POWER OUTPUT: Time-averaged

OWC Dynamics

Stochastic model:

- Linear turbine (Wells turbine)
- Random Gaussian waves



Pierson-Moskowitz spectrum $S_\zeta(\omega) = 263\, H_s^2 T_e^{-4} \omega^{-5} \exp(-1054\, T_e^{-4}\omega^{-4})$.

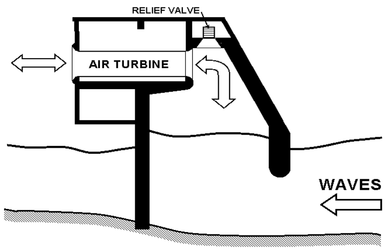
For linear system, $p(t)$ is random Gaussian, with variance

$$\sigma_p^2 = \int_0^\infty S_\zeta(\omega) \left|\Gamma(\omega)\Lambda(\omega)\right|^2 d\omega \quad \text{where} \quad \Lambda = \left[\left(\frac{KD}{\rho_a N} + B\right) + i\left(\frac{\omega V_0}{\rho_a c_a^2} + C\right)\right]^{-1}$$

and pdf $f(p) = \dfrac{1}{\sqrt{2\pi}\,\sigma_p} \exp\left(-\dfrac{p^2}{2\sigma_p^2}\right)$

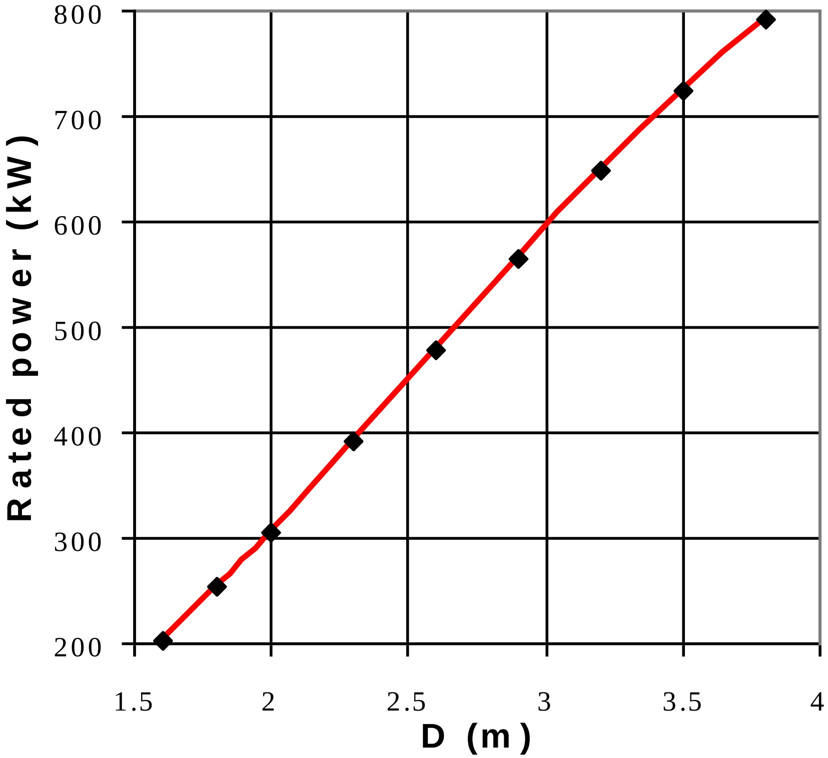
$$\overline{P_t} = \int_{-\infty}^{\infty} f(p)\, P_t(p)\, dp = \frac{2\rho_a N^3 D^5}{\sqrt{2\pi}\,\sigma_p} \int_0^\infty \exp\left(-\frac{p^2}{2\sigma_p^2}\right) f_P\left(\frac{p}{\rho_a N^2 D^2}\right) dp$$

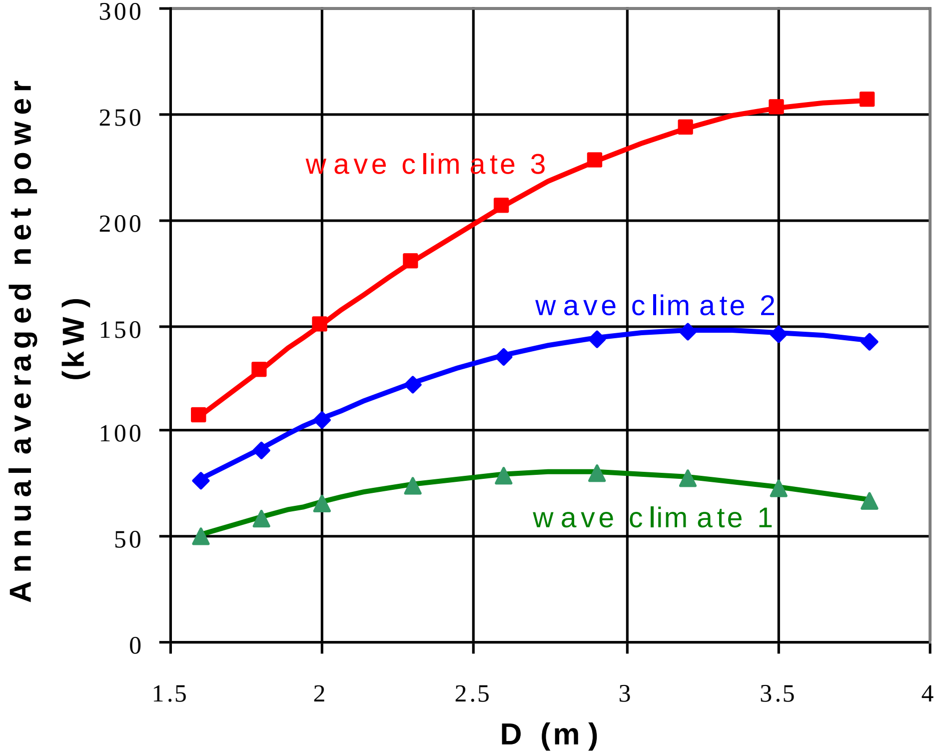
OWC Dynamics



Application of stochastic model

Wells turbine size range 1.6m < *D* < 3.8m





Plant rated power
(for H_s = 5m, T_e=14s)

Annual averaged
net power (electrical)

Model Testing

Theoretical/numerical modelling, based on linear water wave theory, is unable to account for:

- **large amplitude waves**
- **large amplitude motions of bodies and OWCs**
- **real-fluid effects (viscosity, turbulence, eddies)**
- **Survival in very energetic seas**

Model testing in wave tank is essential to:

- **validate theoretical results**
- **investigate non-linear effects effects**
- **investigate survival issues**

It is an essential step before testing under real sea conditions.

Model Testing

How to scale up model test results, assuming geometric similarity ?

Dimensionless coefficients

L	**Linear dimension**
$\dfrac{T}{g^{-1/2} L^{1/2}}$	**Time (wave period)**
$\dfrac{V}{(gL)^{1/2}}$	**Velocity**
$\dfrac{F}{\rho g L^3}$	**Force**
$\dfrac{P}{\rho g L^{7/2}}$	**Power**

Model Testing

Required testing conditions:

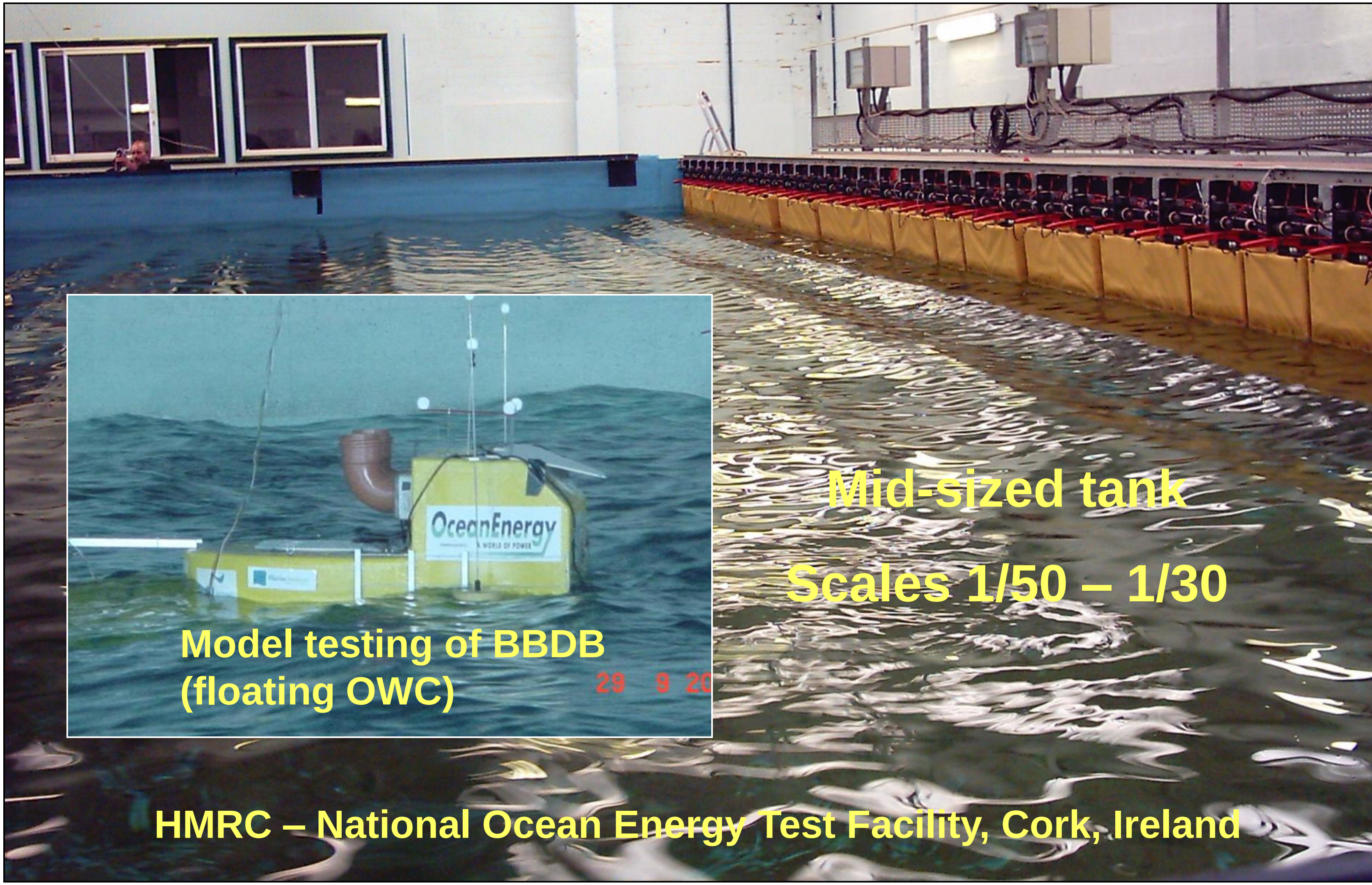
- **For power performance: H_S up to 4 - 5 m.**
- **For survival H_S up to 10 - 15 m (individual wave height 18 – 28 m ?) for offshore devices.**
- **Tank depth may be important for simulation of mooring systems.**

Testing for survival is usually done at smaller scale than for power performance, due to limitations in wave generation by wave makers.

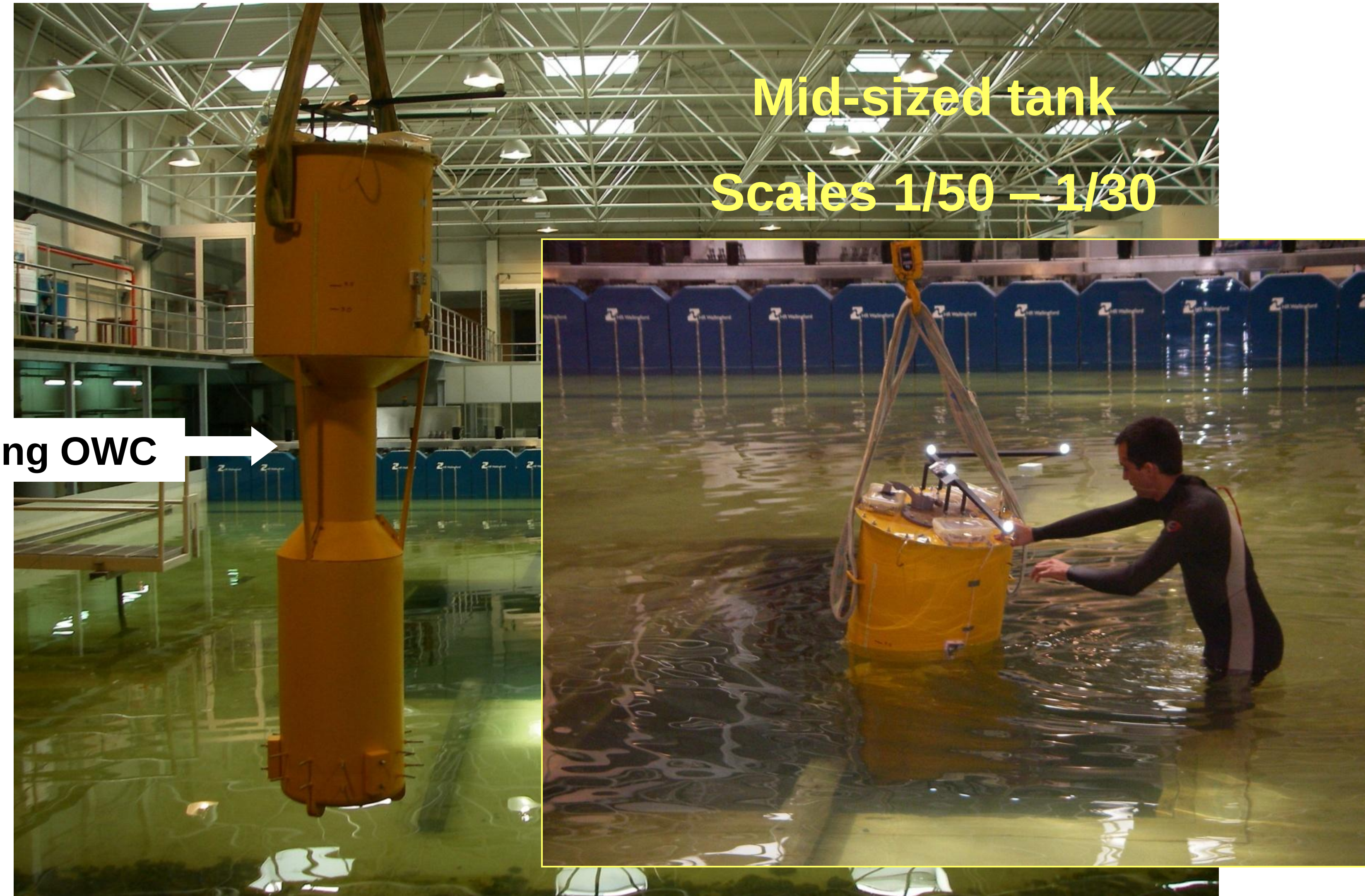
Typical scales for power performance testing:

- **1/80 to 1/25 in small to medium tanks.**
- **Up to 1/10 in very large tanks.**

Model Testing

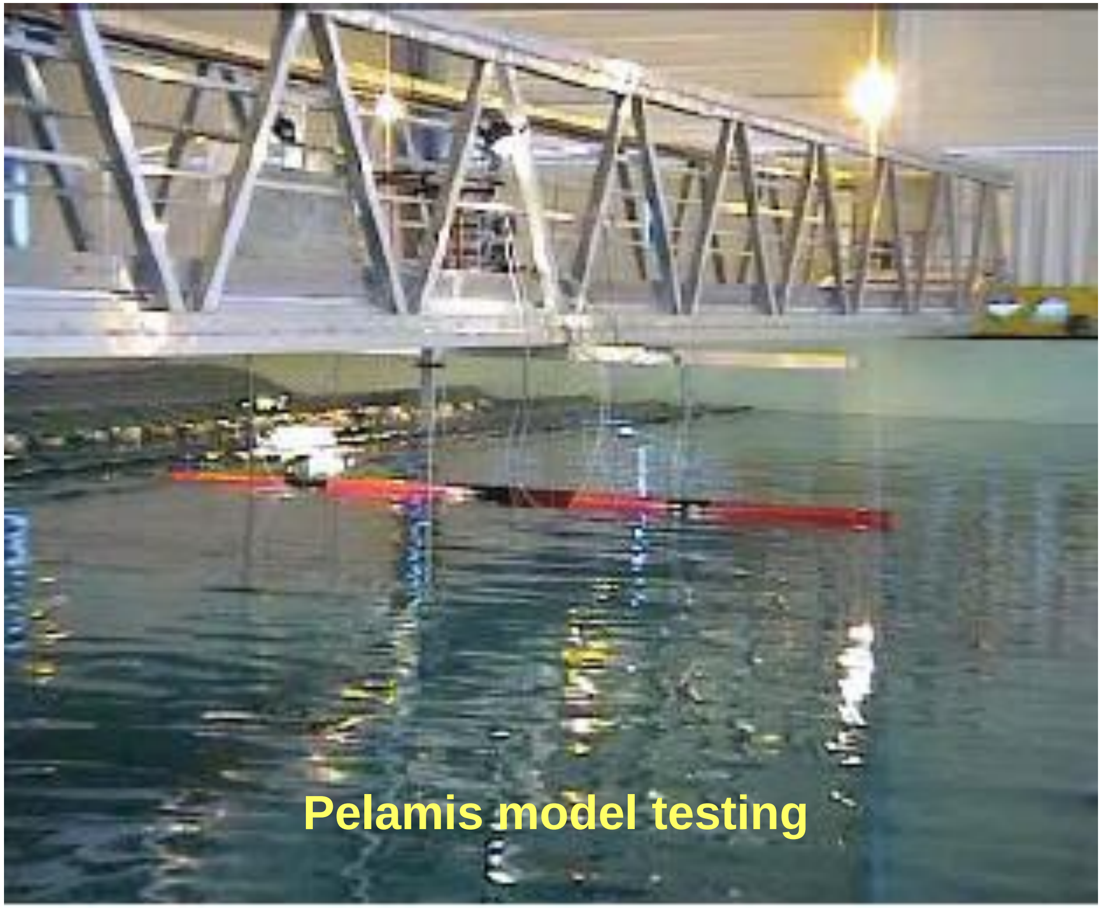


Model Testing



Model Testing

Large tank: Ecole Centrale de Nantes, France


Pelamis model testing

Model Testing

$$\frac{P}{\rho\, g L^{7/2}}$$

What is the power level in the simulation of a 500 kW full-sized prototype ?

Scale	Power in model
1/50	0.6 W
1/25	6.4 W
1/15	38 W
1/10	0.16 kW
1/4	3.9 kW

A realistic simulation of the PTO (hydraulic, linear generator, etc.) with control capability in general requires scale larger than 1/10.

Some technology developers use tests at scales 1/4 to 1/5 in sheltered sea conditions.

Model Testing

OWC testing

- **The spring-like effect of air compressibility in the air chamber is important.**

- **In testing, the air chamber volume should be scaled as $L^{1/2}$ not as $L^{1/3}$.**

- **This may raise practical problems, especially when model testing floating OWCs.**

- **The presence of the air turbine is usually simulated by a pressure drop.**

- **There are several techniques for doing that (orifice, porous plate, …).**

END OF PART 3
WAVE ENERGY CONVERSION MODELLING