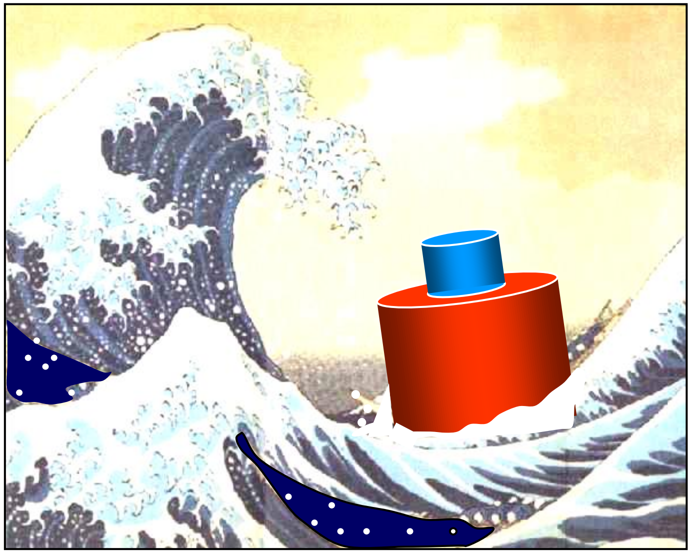
Università degli Studi di Firenze, 18-19 April 2012



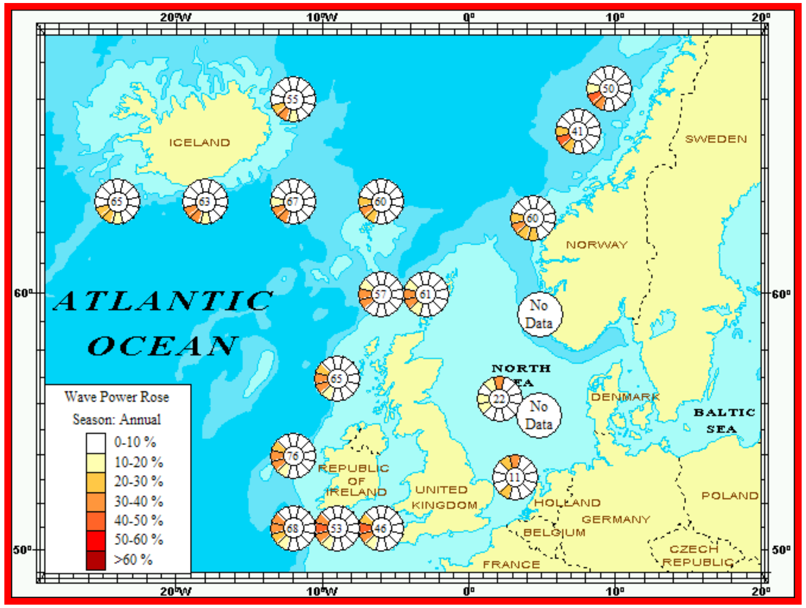
WAVE ENERGY UTILIZATION

António F. O. Falcão

Instituto Superior Técnico,
Universidade Técnica de Lisboa

Part 1

Wave Energy Resource



WAVE ENERGY

SOLAR ENERGY

↓

WIND ENERGY

↓

WAVE ENERGY

Typical values of wave energy flux (annual average):

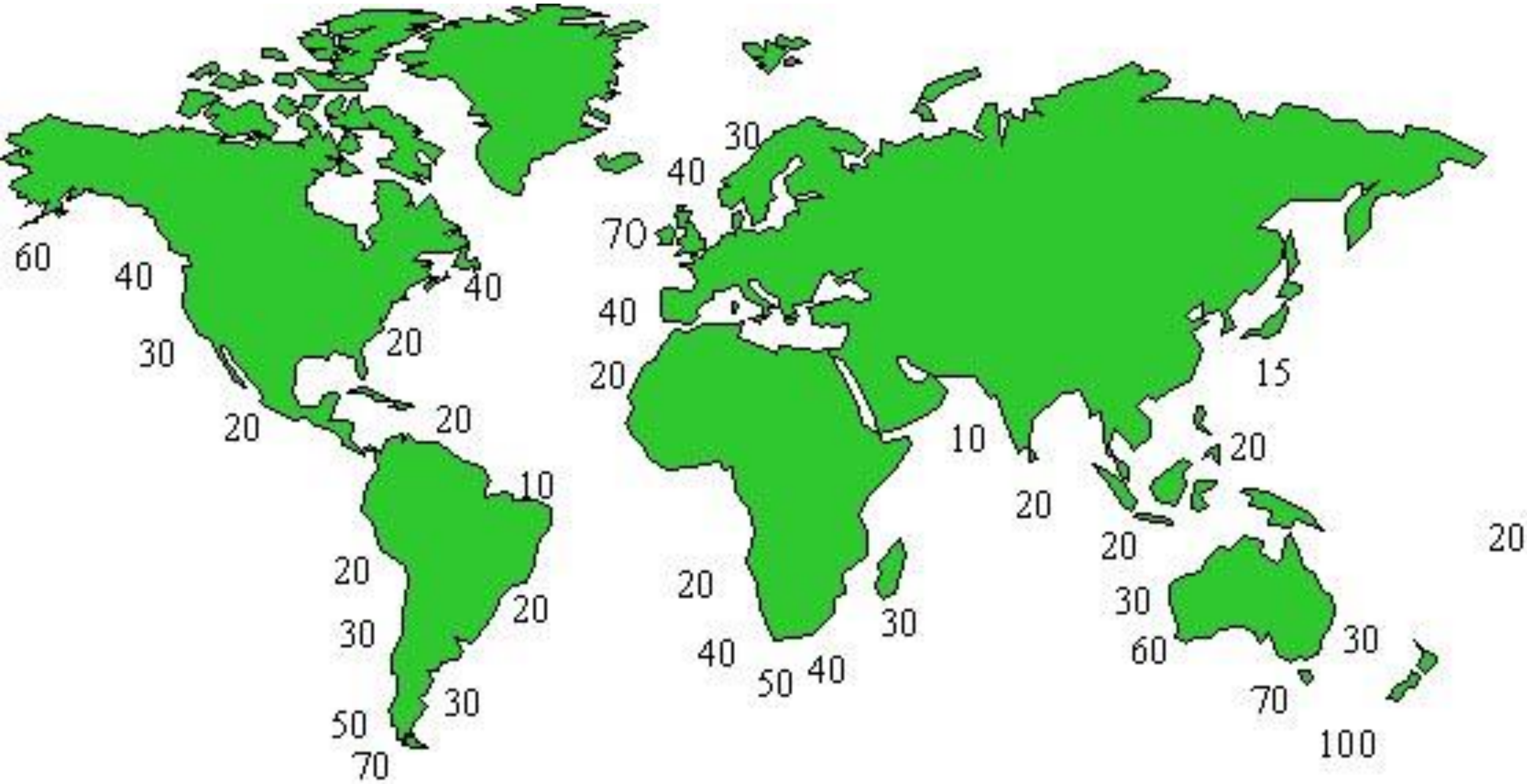
Deep water: 6-70 kW/m

Near shore: lower values,
Depending on:

- bottom slope
- local depth (wave breaking)
- bottom roughness (friction)
- bottom configuration (diffraction, refraction)

Close to the surface (h<20m): density flux of energy (kW/m^2) much higher than wind energy

World distribution of wave energy level
Annual-averaged values in kW/m (deep water, open sea)



THE WAVES AS ENERGY RESOURCE

The waves are generated by the wind.

In deep water (> 100 - 200m) they travel large distances (thousands of km) practically without dissipation.

The characteristics of the waves (height, period, etc.) depend on:

- **Sea surface area acted upon by the wind: "fetch"**
- **Duration of wind action**

"Swell": wave generated at a long distance (mid ocean).

"**Wind sea**": waves generated locally.

In general, swell is more energetic than wind sea.

FLUID MOTION IN WAVES

- **Perfect fluid (no viscosity)**
- **Incompressible flow** $\nabla \cdot \mathbf{V} = 0$
- **Irrotational flow** $\nabla \times \mathbf{V} = 0$ or $\mathbf{V} = \nabla \phi$

Laplace equation $\nabla^2 \phi = 0$

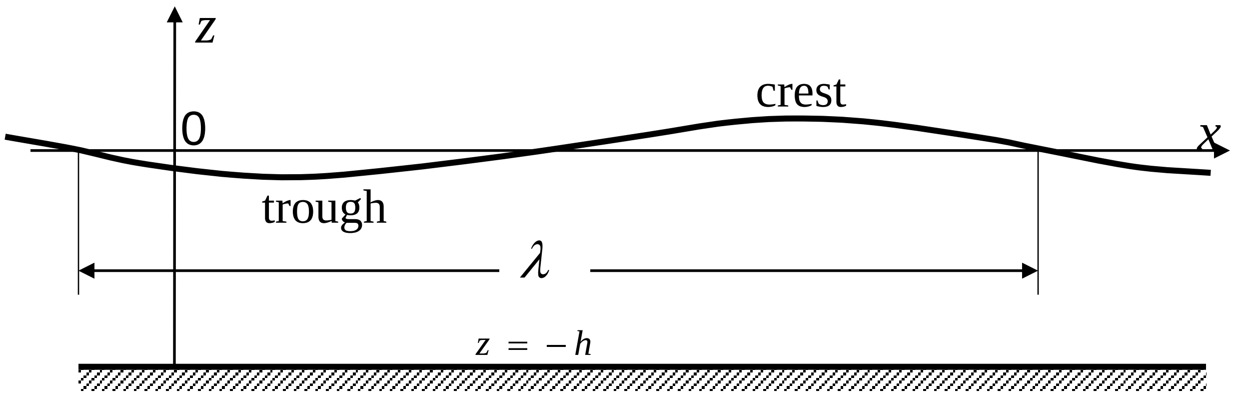
Boundary conditions

- At the free-surface: $p = p_{at}$
- At the bottom: $\mathbf{V} \cdot \mathbf{n} = 0$

- **The free-surface is unknown, which makes the problem non-linear.**

In general the boundary condition is applied at the undisturbed free-surface (flat surface): LINEAR THEORY.

The simplest solution: the **sinusoidal regular wave**



$$\phi = \text{const} \times \cosh\left[\frac{2\pi}{\lambda}(z+h)\right]\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x + \alpha_0\right)$$

If $h = \infty$

$$\phi = \text{const} \times \exp\left(\frac{2\pi}{\lambda}z\right)\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x + \alpha_0\right)$$

T = period (s), $f = 1/T$ = frequency (Hz or c/s),

$\omega = 2\pi f = 2\pi / T$ = radian frequency (rad/s),

λ = wavelength (m), $k = 2\pi/\lambda$ = wave number (m^{-1})

Free-surface elevation

$$\zeta(x,t) = A\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x + \alpha_0\right) = A\sin(\omega t - kx + \alpha_0)$$

A = **wave amplitude**

$H = 2A$ = **wave height** (from trough to crest)

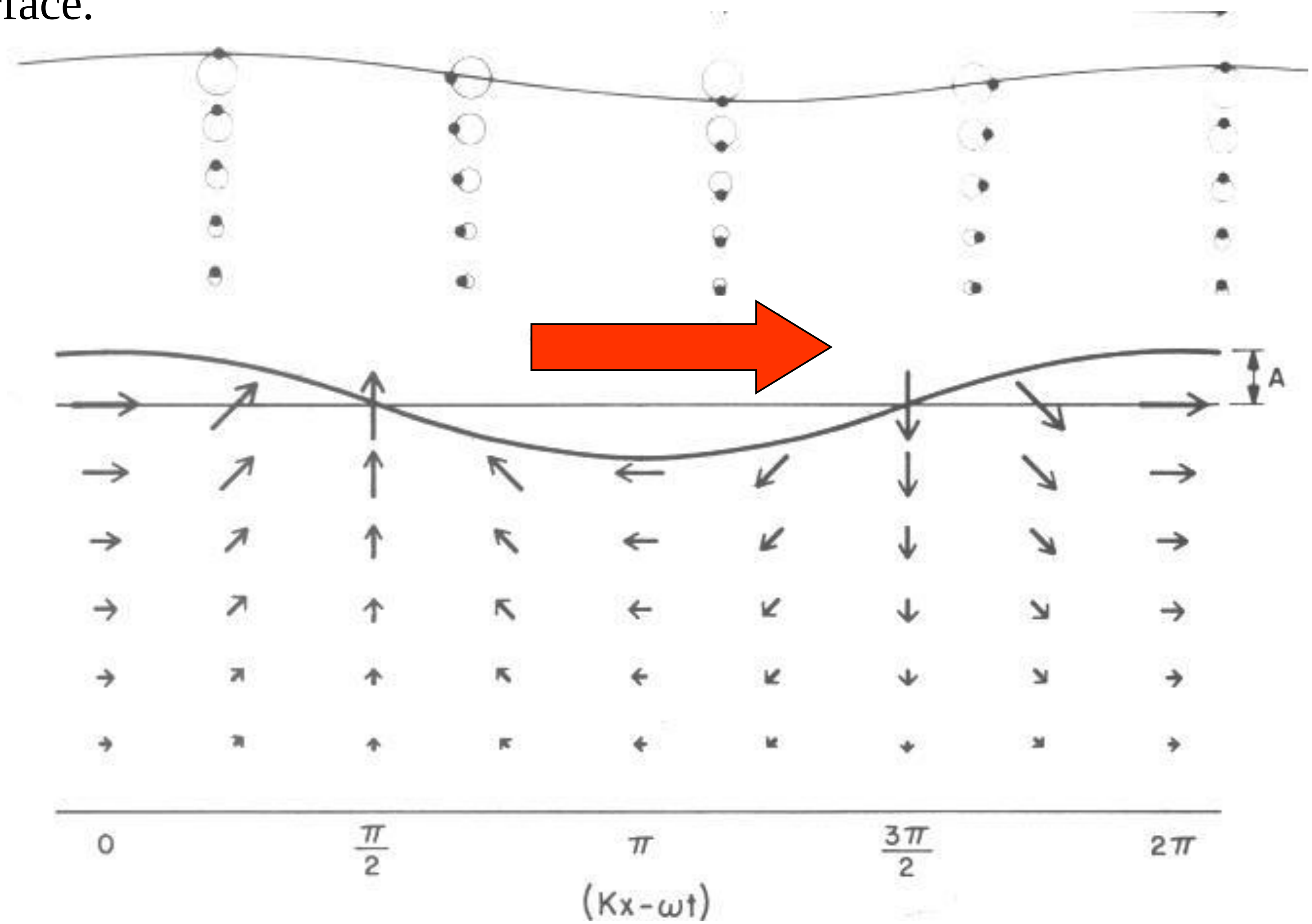
$$\phi = \text{const} \times \exp\left(\frac{2\pi}{\lambda}z\right)\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x + \alpha_0\right)$$

The disturbance decreases with the distance to the surface.

In deep water, the decrease is exponential: the disturbance practically vanishes at a depth of about 1/2 wavelength.

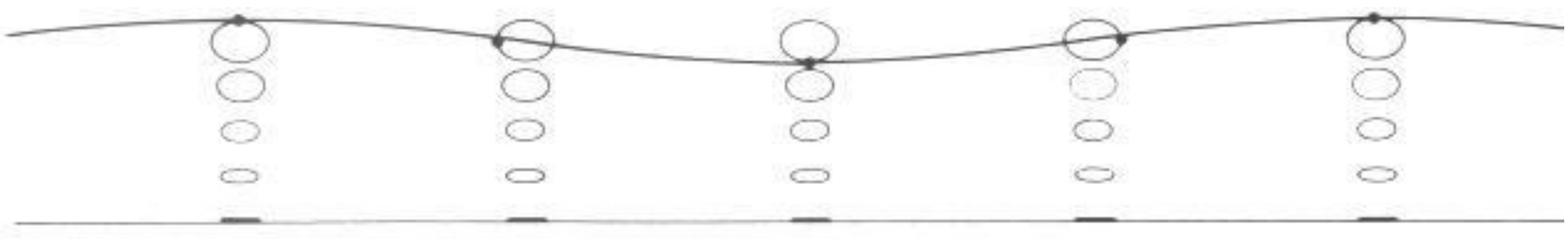
In **deep water**, the water particles have **circular orbits**.

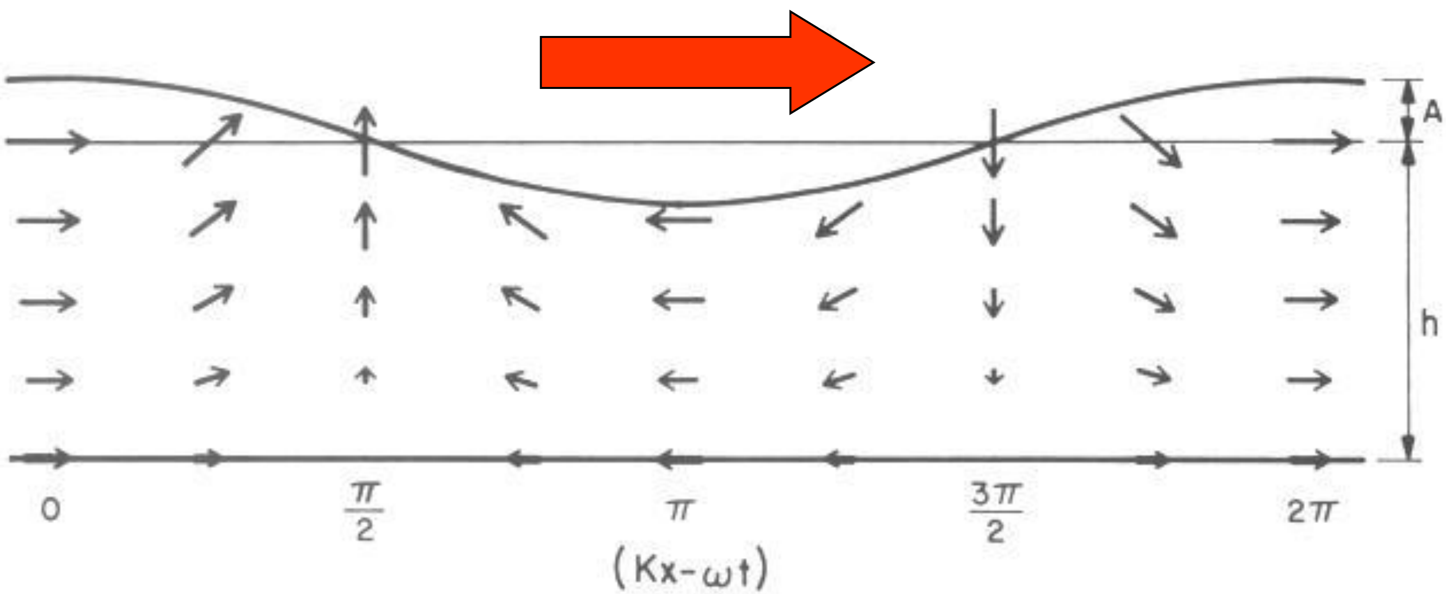
The orbit radius **decreases exponentially** with the distance to the surface.



In water of **finite depth**, the orbits are **ellipses**.

The ellipses become flat near the bottom.





Propagation velocity (phase velocity)

$$\boxed{c = \frac{\lambda}{T} = \frac{\omega}{k}}$$

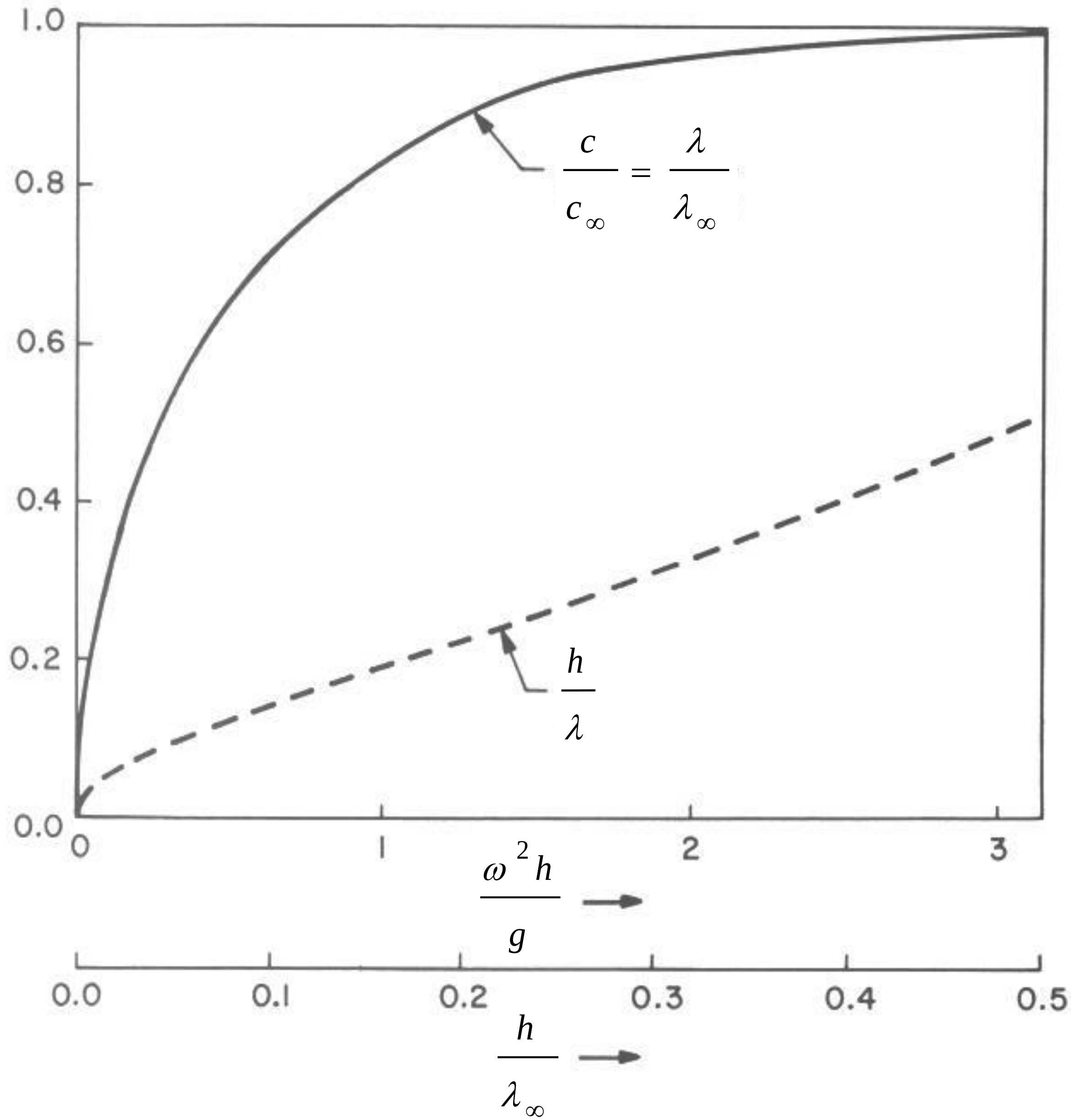
From the boundary condition at the sea surface:

$$c\,\frac{\omega}{g} = \tanh\,\frac{\omega\,h}{c}$$

The velocity of propagation c depends on the wave period T (or frequency ω or f) and also on the water depth h.

The sea is a dispersive medium for surface waves.

The speed of sound in air is independent of frequency.


1.0
0.8
0.6
0.4
0.2
0.0
$\frac{c}{c_\infty} = \frac{\lambda}{\lambda_\infty}$
$\frac{h}{\lambda}$
0
1
2
3
$\frac{\omega^2 h}{g}$ →
0.0
0.1
0.2
0.3
0.4
0.5
$\frac{h}{\lambda_\infty}$ →

Limiting situations

$$\boxed{c\,\frac{\omega}{g} = \tanh\frac{\omega h}{c}}$$

In deep water (in practice if $h > \lambda/2$) **:** $\tanh\frac{\omega h}{c} = \tanh(kh) \cong 1$

$$c = \sqrt{\frac{g}{k}} = \frac{g}{\omega} = \frac{gT}{2\pi}$$

In shallow water (in practice if $h << \lambda$) $\tanh(kh) \cong kh$

$$c = \sqrt{gh}$$

c does not depend on T

Example

$T = 8 \text{ s} \qquad g = 9{,}8 \text{ m/s}^2$

Deep water $\quad c = \dfrac{gT}{2\pi} = \dfrac{9{,}8 \times 8}{2\pi} = 12{,}5 \text{ m/s} \qquad \lambda = cT = 12{,}5 \times 8 = 100 \text{ m}$

Shallow water $\quad h = 1 \text{ m}$

$$c = \sqrt{gh} = \sqrt{9{,}8 \times 1} = 3{,}1 \text{ m/s} \qquad \lambda = cT = 3{,}1 \times 8 = 25{,}0 \text{ m/s}$$

Intermediate water depth $\quad h = 15 \text{ m} \qquad \omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{8} = 0{,}785 \text{ rad/s}$

$$c \frac{\omega}{g} = \tanh \frac{\omega h}{c} \qquad c \times \frac{0{,}785}{9{,}8} = \tanh \frac{0{,}785 \times 15}{c} \qquad c = 10{,}2 \text{ m/s}$$

$$\lambda = cT = 10{,}2 \times 8 = 81{,}8 \text{ m}$$

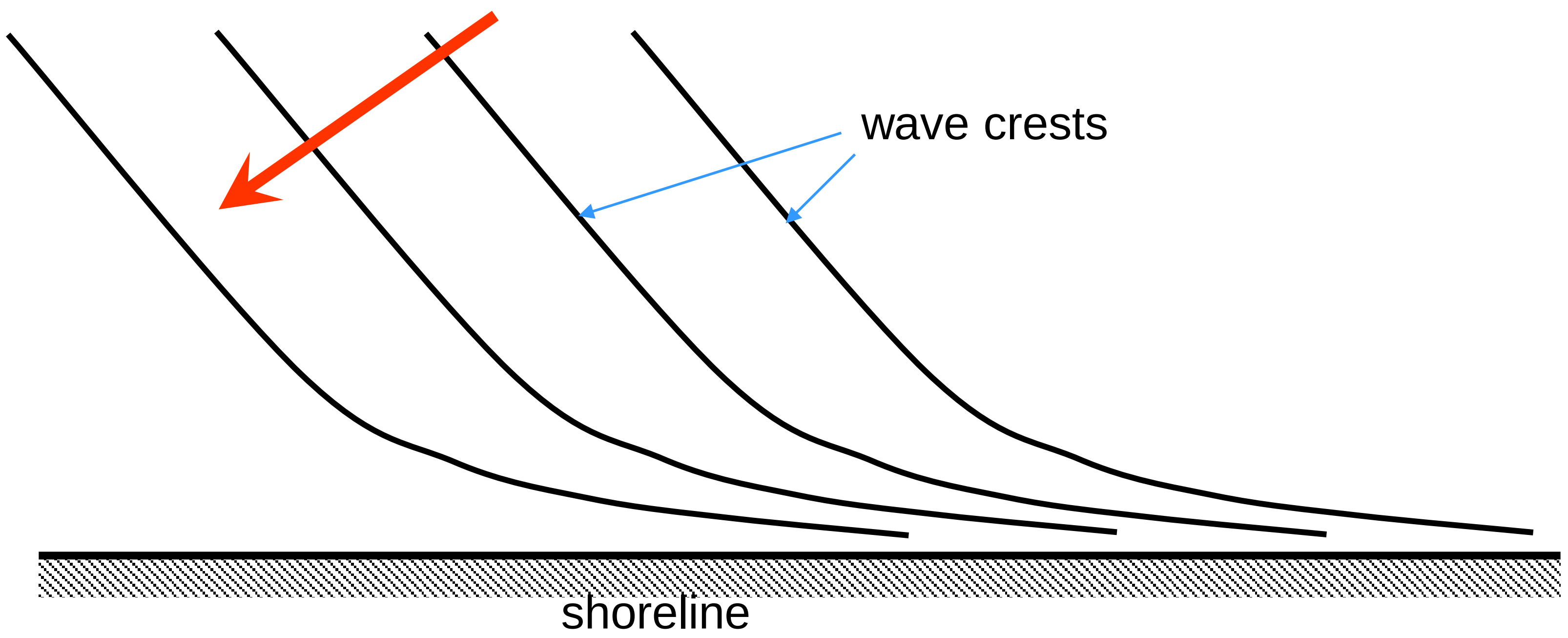
$\boxed{T = 8 \text{ s}}$

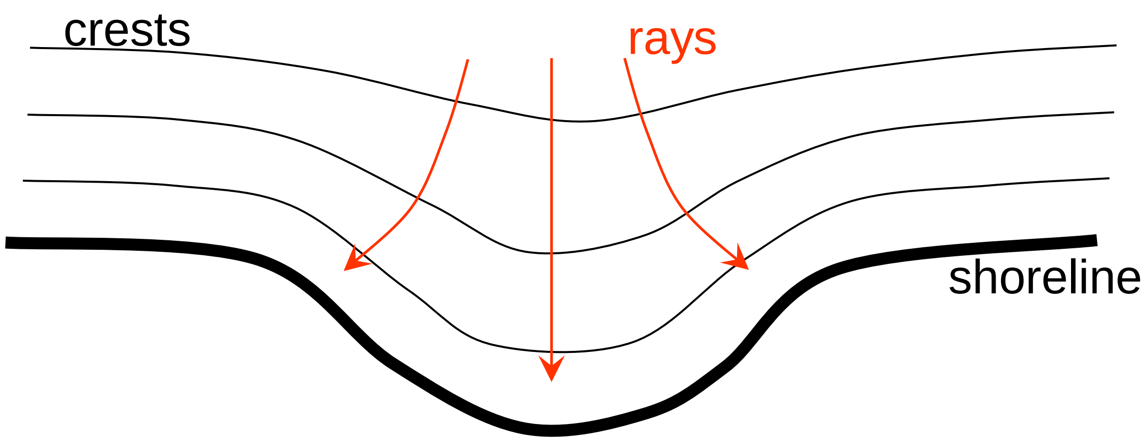
h (m)	c (m/s)	λ (m)
1	3,10	24,8
3	5,25	42,0
5	6,63	53,0
10	8,86	70,9
15	10,22	81,8
20	11,09	88,7
25	11,65	93,2
30	12,00	96,0
40	12,33	98,6
50	12,44	99,5
∞	12,48	99,8

Refraction effects due to bottom bathymetry

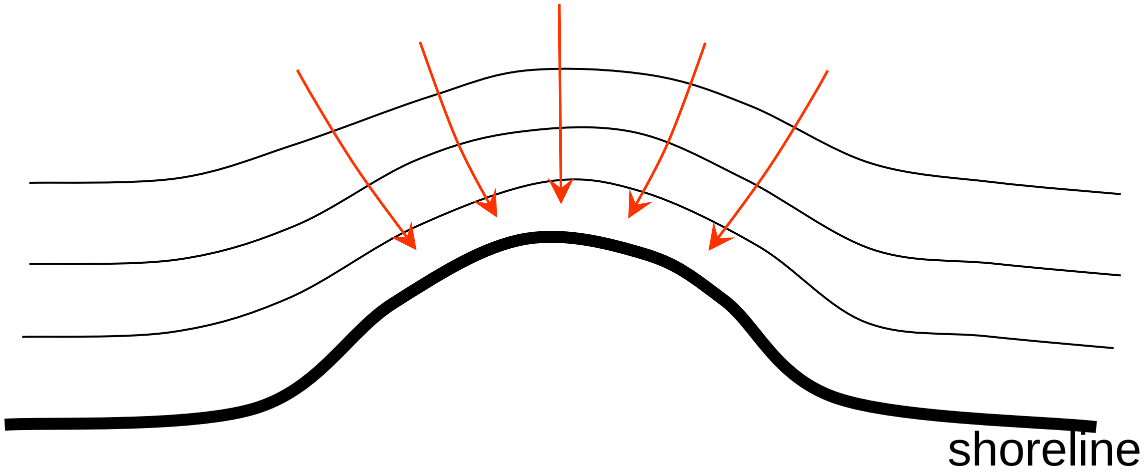
The propagation velocity c decreases with decreasing depth h.

As the waves propagate in decreasing depth, their crests tend to become parallel to the shoreline





Dispersion of energy at a bay.



Concentration of energy at a headland.

Group velocity or velocity of propagation of energy

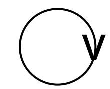
The **velocity of propagation of wave energy**, c_g, is different from (smaller than) the phase velocity or velocity of propagagtion of the crests c.

In deep water, it is $$c_g = \frac{1}{2} c$$

In sound waves, there is no difference between the two velocities.

ENERGY OF THE WAVES

- Kinetic energy (circular or elliptic orbits)



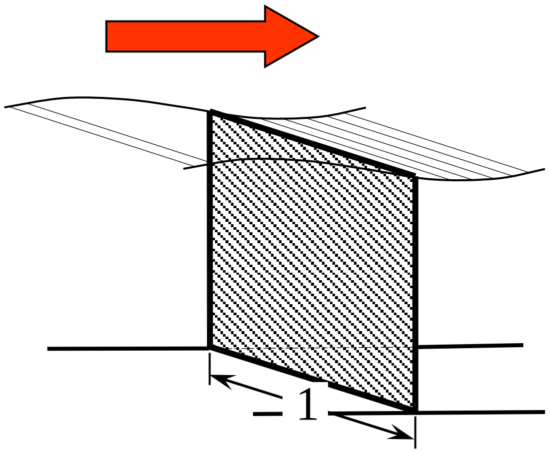
- Potential energy (sea surface is not plane)

In deep water, **energy per unit horizontal area**, time-averaged:

$$W_{kin} = W_{pot} = \frac{1}{4}\rho g A^2 = \frac{1}{16}\rho g H^2 \qquad (H = 2A)$$

$$\boxed{W = W_{kin} + W_{pot} = \frac{1}{2}\rho\, g A^2 = \frac{1}{8}\rho\, g H^2 \qquad (\mathrm{J}/\mathrm{m}^2)}$$

We are more interested in the **energy flux** across a vertical plane parallel to the wave crests (from bottom to surface).



Energy flux per unit length along wave crests (time-averaged)

In deep water

$$\boxed{E = \frac{1}{8\pi}\rho g^2 A^2 T \qquad \text{(W/m)}}$$

Note:

- The energy flux is proportional to the wave period *T* and to the square of the wave amplitude *A* (or the wave height $H = 2A$).
- This is energy flux from surface to bottom.
- Most of the contribution to *E* is from the upper layer close to the sea surface.

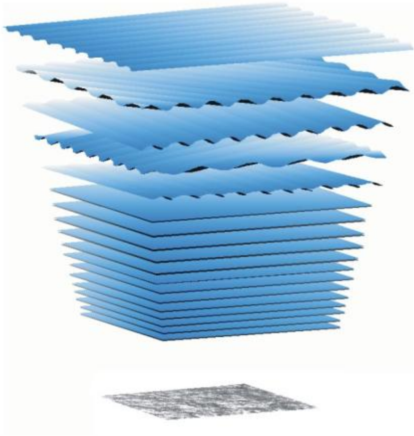
REAL IRREGULAR WAVES

Real waves are not sinusoidal.

However, they can be represented with good approximation as superpositions of sinusoidal (regular) waves.

surface elevation

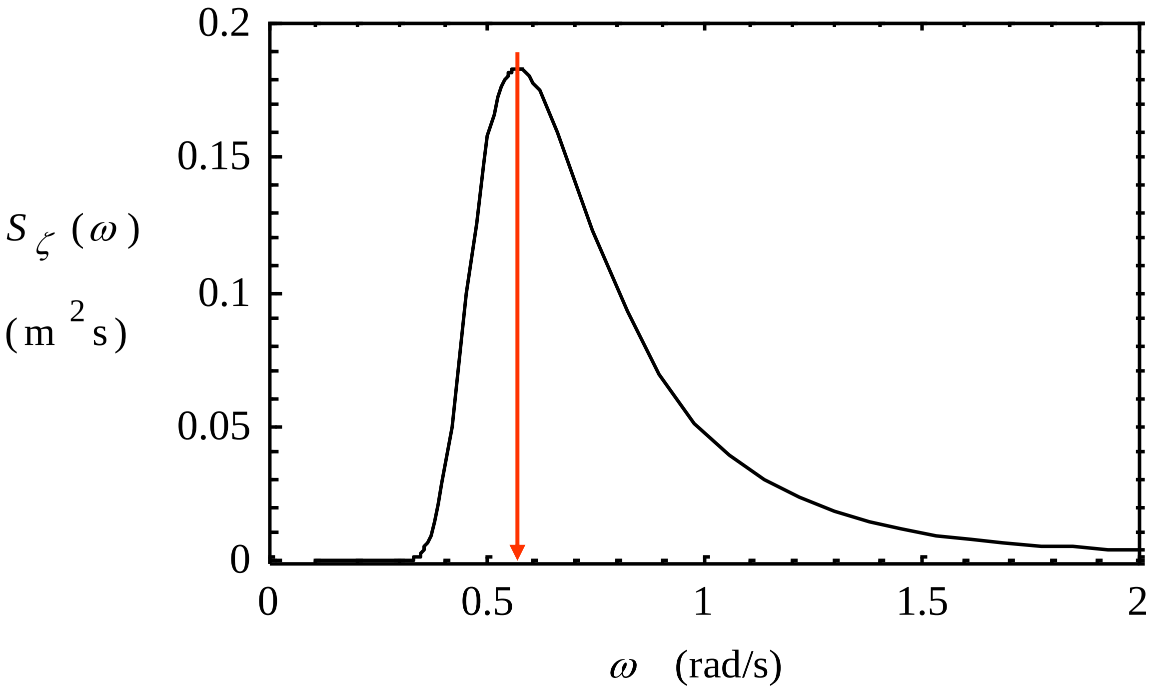
$$\zeta(x,t) = \sum_{n=1}^{N} A_n \sin(\omega_n t - k_n x + \alpha_n)$$



If $N \to \infty$, we have a **continous spectrum**.

Frequently a **power spectum** $S_\zeta(\omega)$ is defined (rather than for **amplitude**).

Example of power spectrum



In practice, for numerical simulations, the spectrum has to be **discretized**

$$\boxed{\zeta(x,t) = \sum_{n=N_{\min}}^{N_{\max}} A_n \sin\left(\omega_n t - k_n x + \alpha_n\right)}$$

$\omega_n = n\omega_0$

k_n corresponding wave number

ω_0 small frequency interval

$A_n = \sqrt{4\omega_0 S_\zeta(n\omega_0)}$

$\alpha_n \quad (0 \le \alpha_n \le 2\pi)$ random phase

Power spectrum

For a given sea state, the power spectrum my be obtained from records of **wave measurements** (surface elevation) and the application of **spectral analysis**.

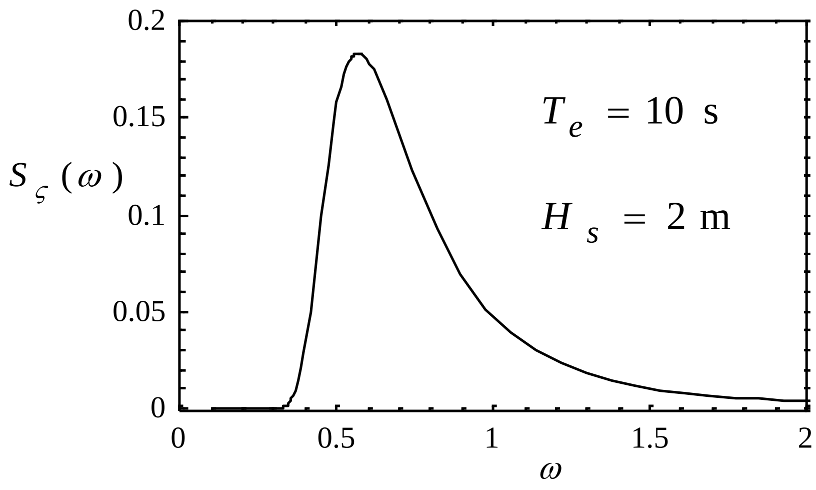
In numerical simulations, spectral distributions are used that fit large classes of sea states.

One is the **Pierson-Moskowitz** spectral distribution:

$$\boxed{S_\zeta(\omega) = 263\, H_s^2 T_e^{-4} \omega^{-5} \exp(-1054\, T_e^{-4} \omega^{-4})}$$

H_s = **significant wave height**

T_e = **energy period**



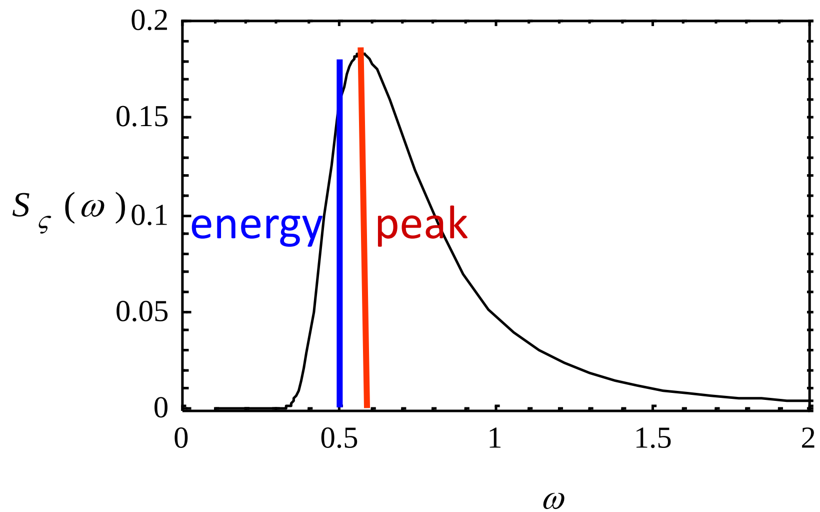
The moments of the wave spectrum

$$\boxed{m_n = \int_0^\infty f^n S_\zeta(f)\,df} \qquad f = \frac{\omega}{2\pi}$$

Significant wave height $\boxed{H_s = 4\sqrt{m_0}}$

$H_s \cong H_{1/3}$ = mean value of the highest 1/3 of wave heights

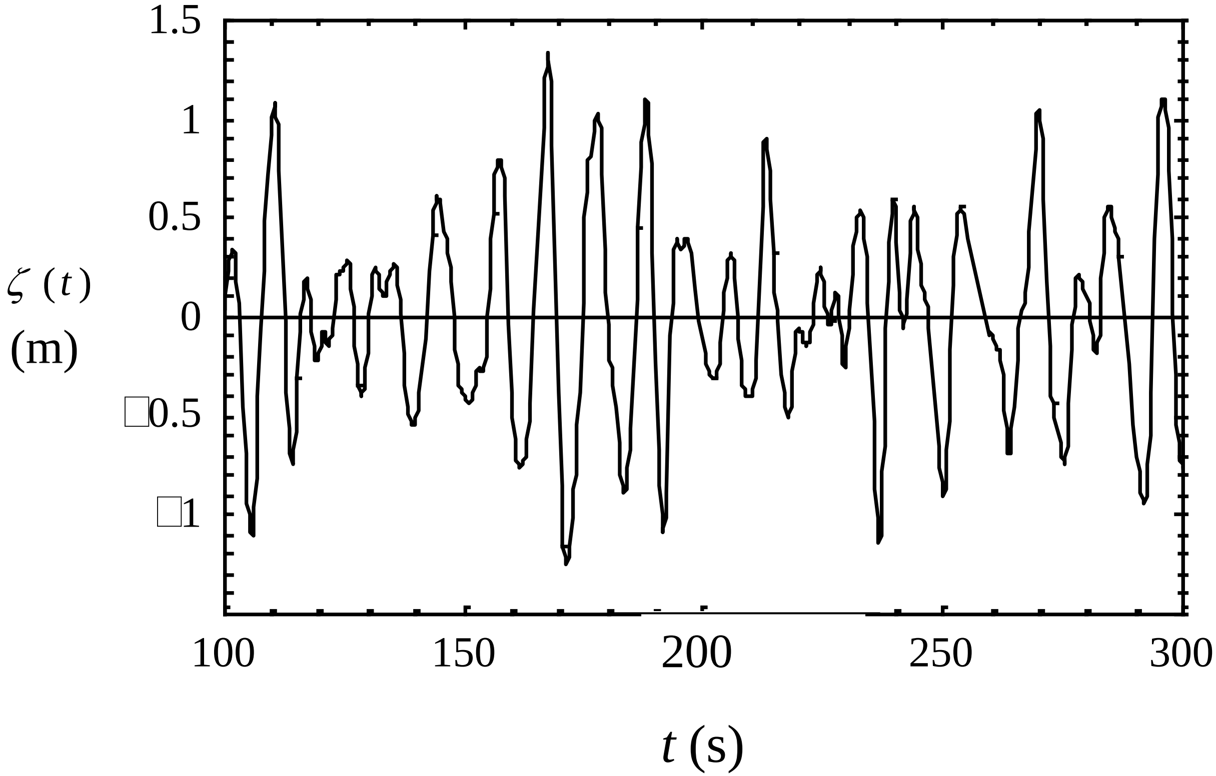
Energy period $\boxed{T_e = \frac{m_{-1}}{m_0}}$



Example

Simulated time-series of **surface elevation** at a given point from a **Pierson-Moskowitz spectrum** discretized into 225 sinusoidal harmonics

$T_e = 10\ \mathrm{s} \qquad H_s = 2\ \mathrm{m}$



Energy flux of irregular waves

If the spectral distribution is known, the energy flux may be obtained as the sumation of the energy fluxes of the sinusoidal harmonics.

For a **Pierson-Moskowitz spectrum**, it is (in deep water)

$$\boxed{E = 0.49\, T_e H_s^2} \qquad \text{(kW/m)}$$

$E\,(\text{kW/m})$ energy flux por unit wave-crest length

$T_e\,(\text{s})$ energy period

$H_s\,(\text{m})$ significant wave height

$$\left\{ \begin{array}{l} T_e = 10\ \text{s} \\ H_s = 3\ \text{m} \end{array} \right. \Rightarrow E = 44.1\ \text{kW/m}$$

Directional spread of the waves

As real waves are not generated at a single point on the ocean, their direction θ is not well defined: there is a **directional spread**.

This applies to a sea state or to a (annual-averaged) wave climate.

A **two-dimensional spectrum** may be defined :

$$S_{\zeta}(\omega, \theta)$$

Cosine law is frequently used: $\boxed{\cos^{2s}(\theta - \theta_0)}$

Larger exponent s means more concentrated directional spectrum.

Wave and Resource Statistics

The **wave climate** may be regarded as a **set of sea states**, each sea state (*i*,*j*) characterized by

- Significant wave height $H_{s,i}$
- Mean energy period $T_{e,j}$
- Frequency of occurrence $F_{i,j}$

$$\boxed{\sum_{i,j} F_{i,j} = 1}$$

Scatter diagram

$T_{e,j}$ / $H_{s,i}$	<4s	4-5m	5-6s	6-7s	7-8s	...
<0.5 m						
0.5-1m						
1-1.5m						
1.5-2m				$F_{i,j}$		
2-2.5m						
2.5-3m						
....						

Wave and Resource Statistics

Annual Relative Frequency in terms of (H_s,T_e)

Off West Portugal, h = 100m, all directions

Hs(m)/Te(s)	< 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9	9 - 10	10 - 11	11 - 12	12 - 13	13 - 14	14 - 16	16 - 18	18 - 20	> 20	
< 0.5																0
0.5 - 1		2	17	31	32	12	8	3	1							105
1 - 1.5		1	11	32	54	26	34	14	3	1	1					176
1.5 - 2			3	21	47	40	31	26	11	5		1				184
2 - 2.5			0	8	29	32	29	32	23	4	3	1				162
2.5 - 3				1	13	29	28	22	17	13	4	3				130
3 - 3.5					7	19	27	18	13	11	7	4				105
3.5 - 4					1	5	14	11	9	8	4	1				53
4 - 4.5					0	3	11	5	6	6	4	1				37
4.5 - 5						1	3	1	4	4	4	3				20
5 - 6								0	2	4	7	5	1			18
6 - 7									1	2	1	5				8
7 - 8									0			1	0			1
8 - 9																0
9 - 10																0
10 - 11																0
11 - 12																0
> 12																0
	0	3	31	93	182	168	185	132	91	57	33	26	1	0	0	1000

Maximum frequency of occurrence -

Wave and Resource Statistics

Annual Energy Distribution in terms of (H_s ,T_e)

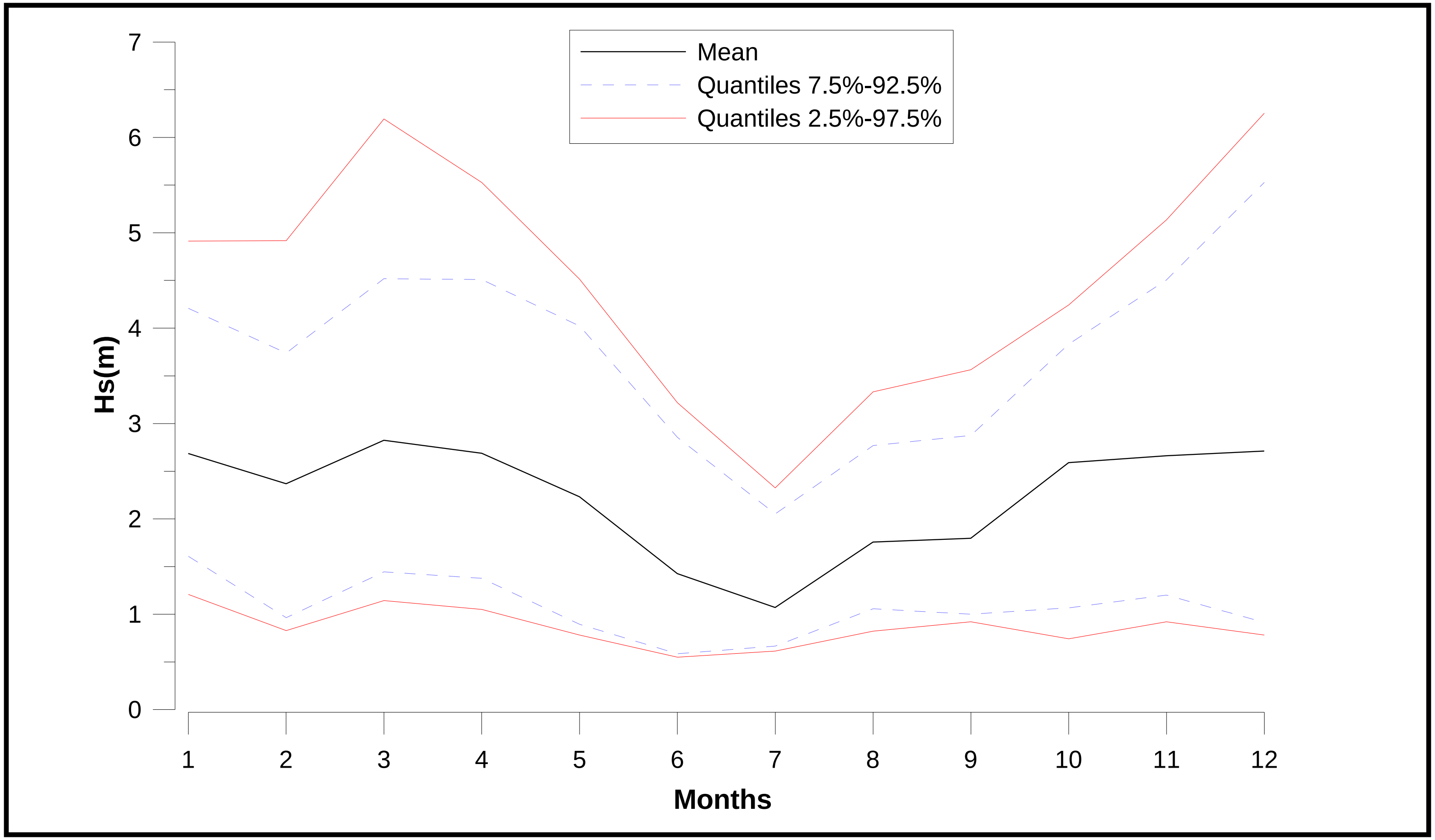
Off West Portugal, h = 100m, all directions

Hs(m)/Te(s)	< 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9	9 - 10	10 - 11	11 - 12	12 - 13	13 - 14	14 - 16	16 - 18	18 - 20	> 20
< 0.5															
0.5 - 1			1	2	1	1	1	0	0						
1 - 1.5				1	7	5	7	5	2	1	0				
1.5 - 2				1	5	9	13	17	8	4		1			
2 - 2.5				0	5	12	14	40	35	3	2	2			
2.5 - 3					3	10	22	33	39	28	5	3			
3 - 3.5						2	37	19	20	37	29	14			
3.5 - 4						2	14	16	23	28	17	11			
4 - 4.5							9	10	20	18	23	7			
4.5 - 5								3	10	16	25	23			
5 - 6									16	34	21	37	7		
6 - 7									6	28		52			
7 - 8									8				11		
8 - 9															
9 - 10															
10 - 11															
11 - 12															
> 12															
													1000		

Maximum energy contribution -

Wave and Resource Statistics

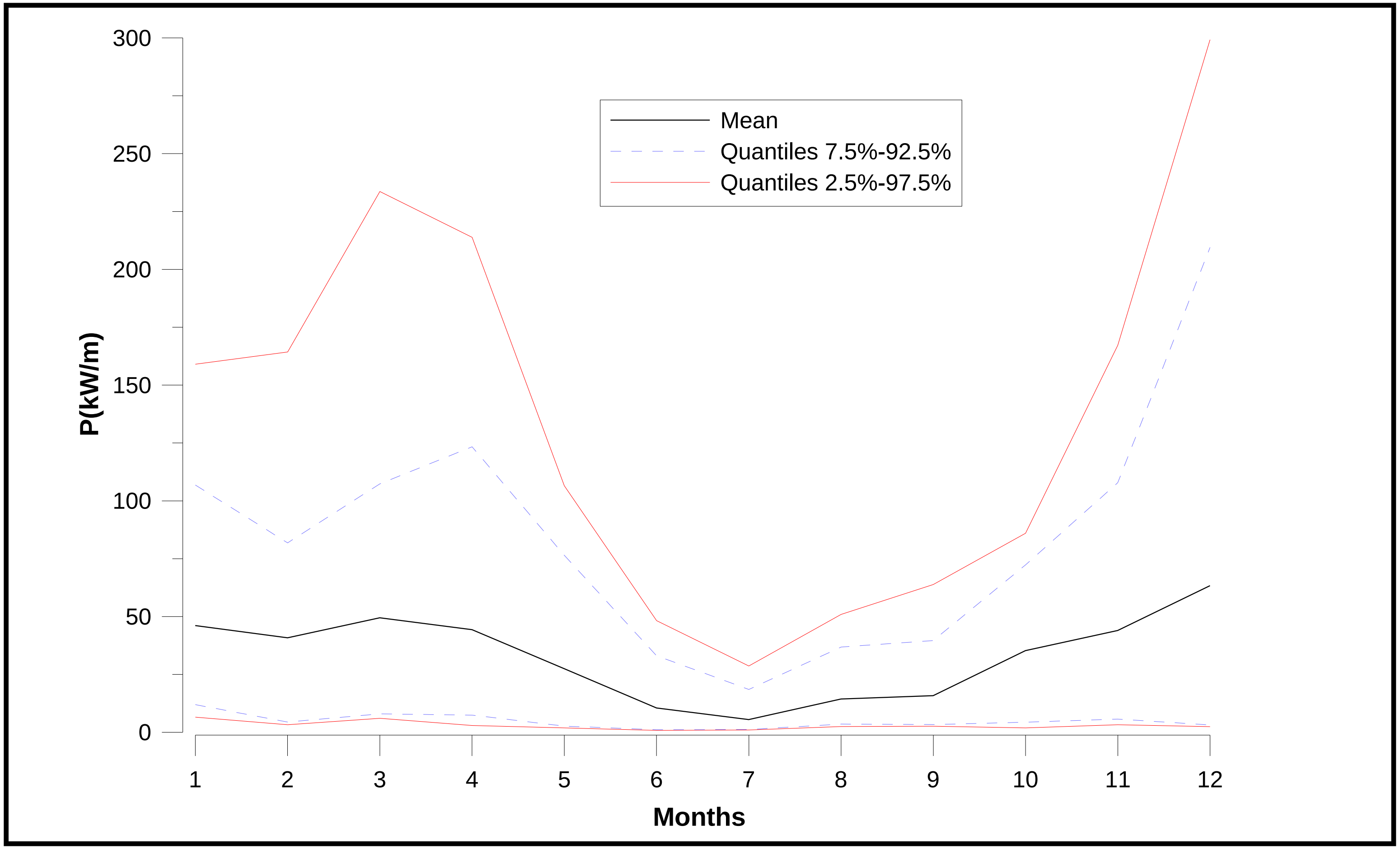
H_s monthly variation- 39ºN - Lisbon



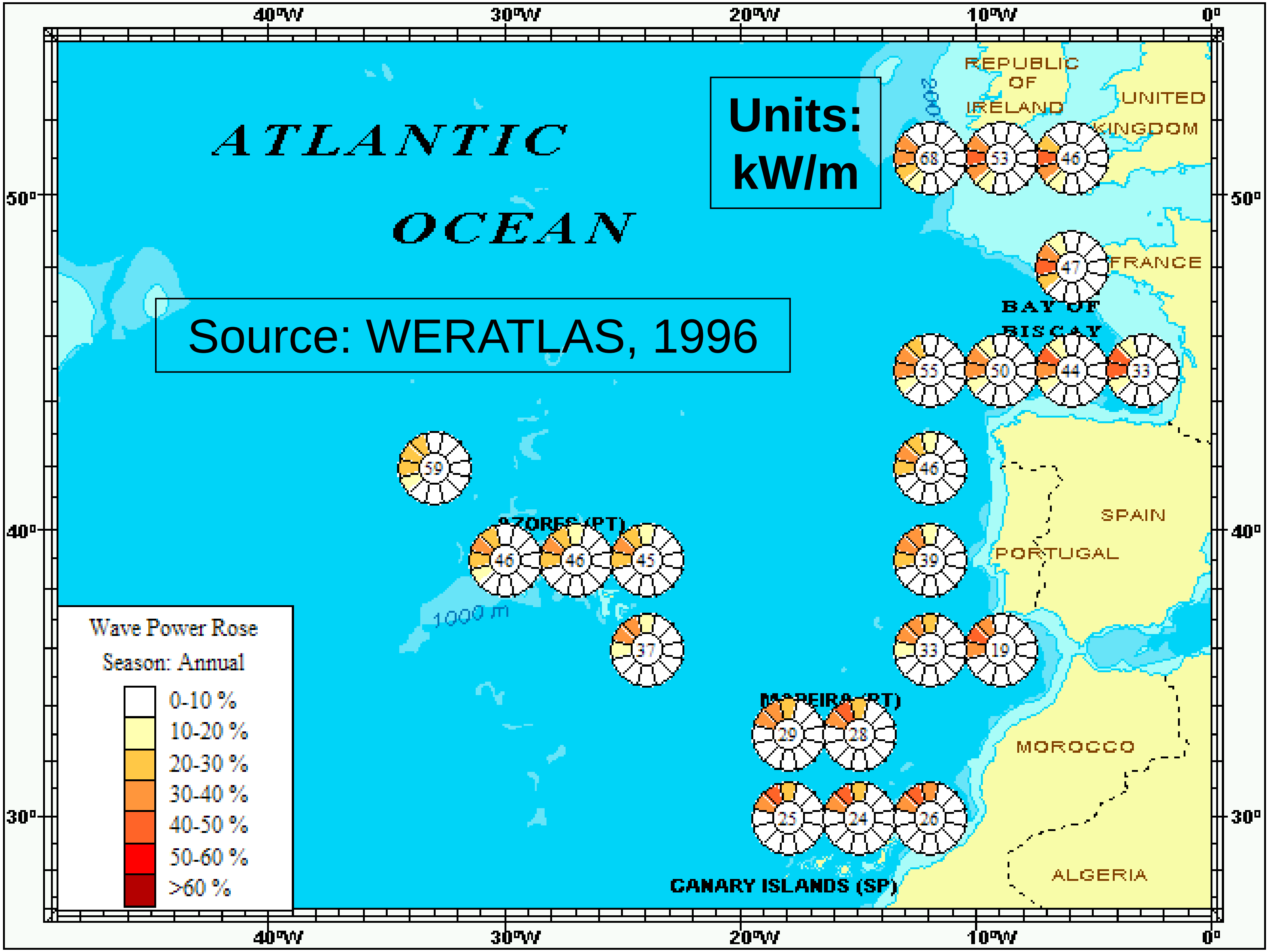
From: WERATLAS

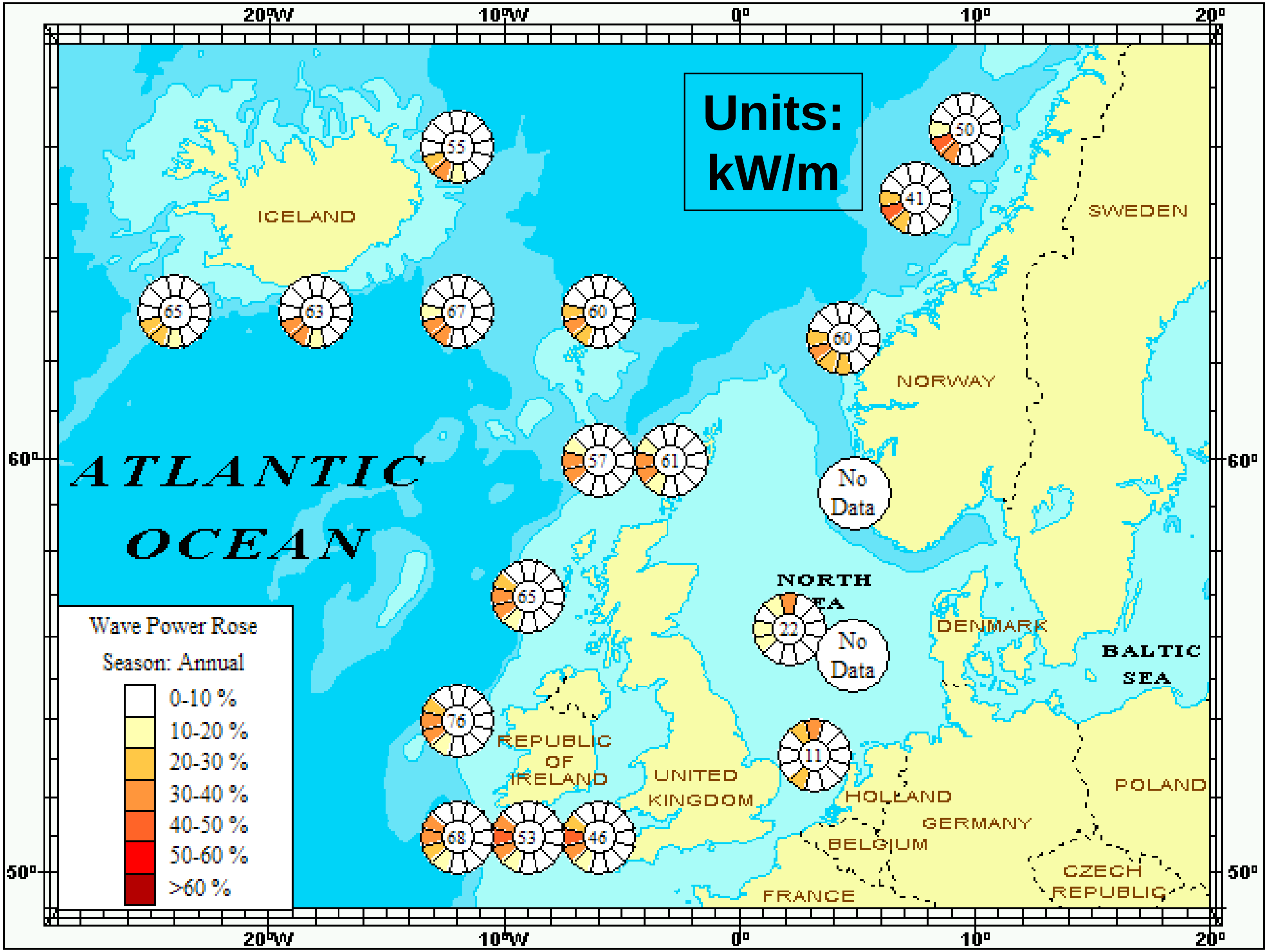
Wave and Resource Statistics

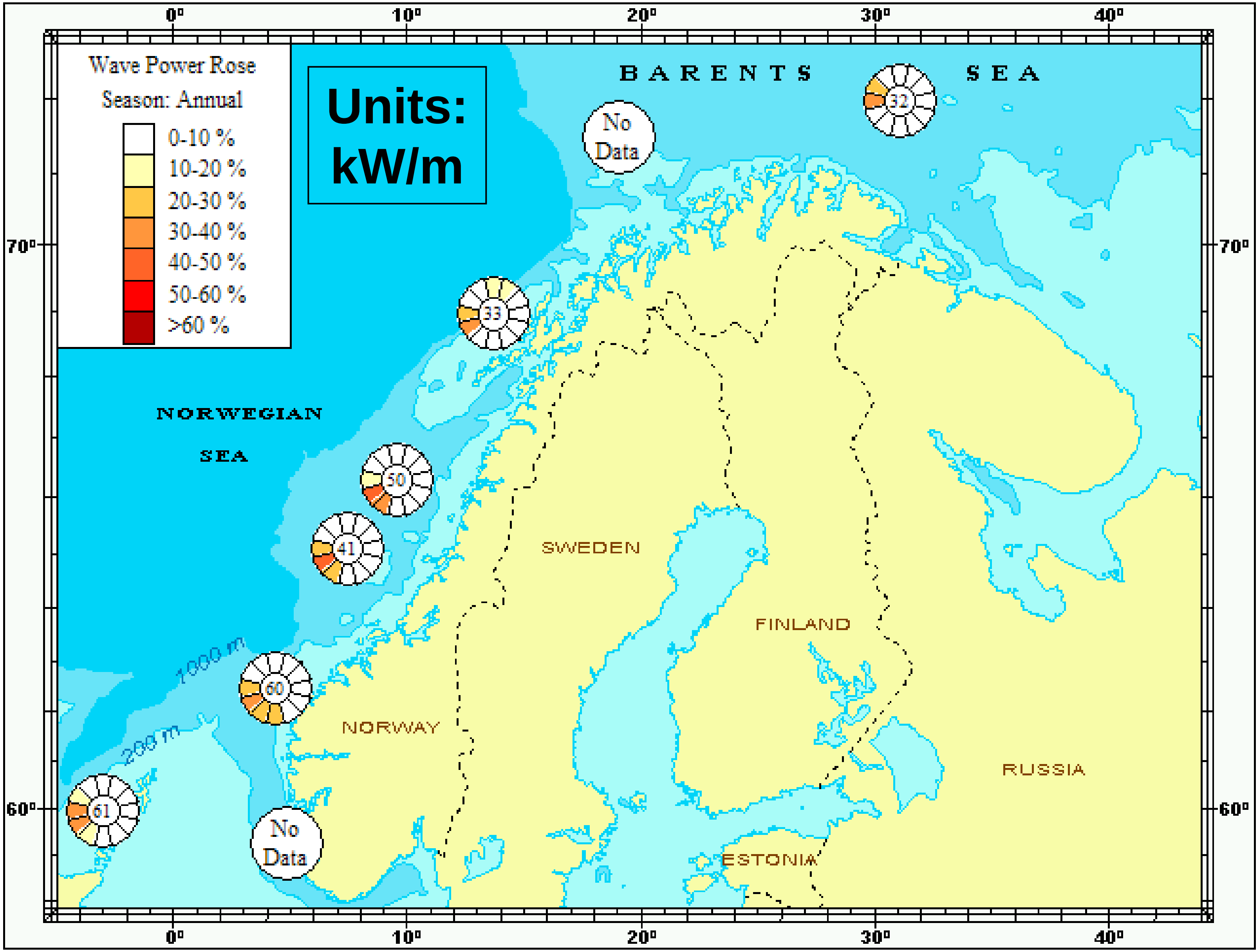
Power Monthly Variation

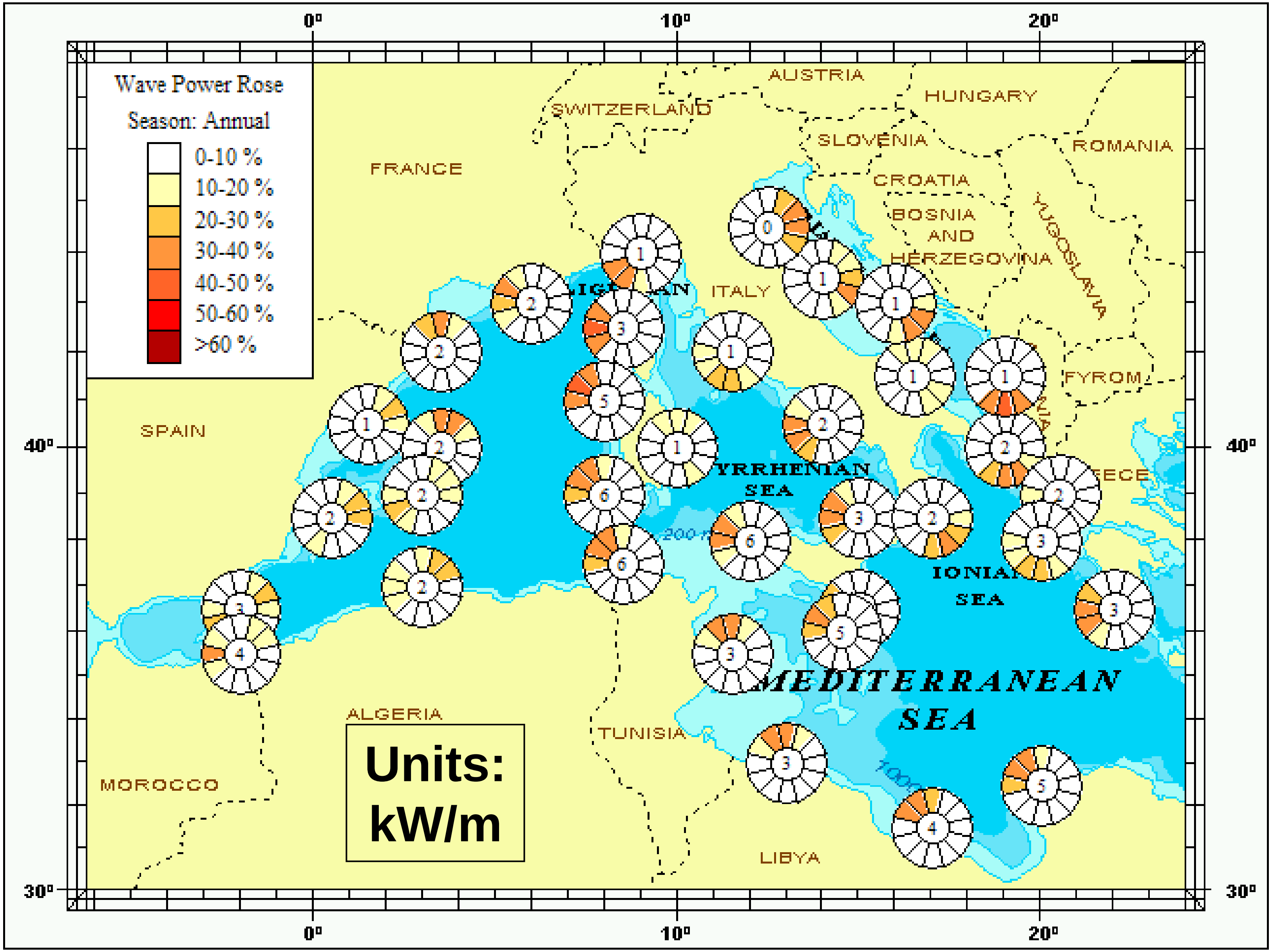


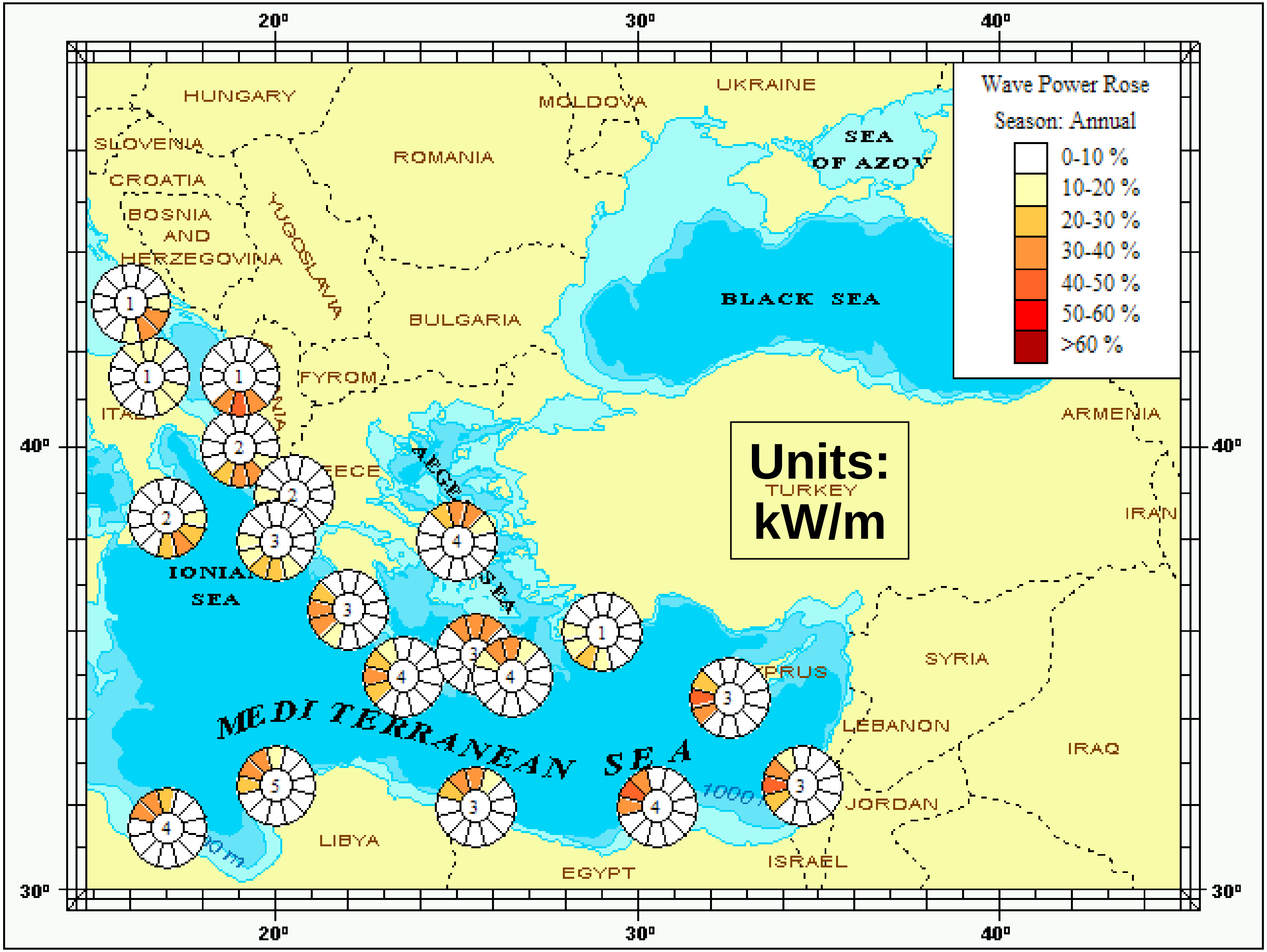
From: WERATLAS


40°W
30°W
20°W
10°W
0°
50°
40°
30°
ATLANTIC
OCEAN
Units:
kW/m
Source: WERATLAS, 1996
REPUBLIC OF IRELAND
UNITED KINGDOM
FRANCE
BAY OF BISCAY
SPAIN
PORTUGAL
MOROCCO
ALGERIA
AZORES (PT)
MADEIRA (PT)
CANARY ISLANDS (SP)
1000 m
200
68
53
46
47
55
50
44
33
59
46
46
46
45
39
37
33
19
29
28
25
24
26
Wave Power Rose
Season: Annual
0-10 %
10-20 %
20-30 %
30-40 %
40-50 %
50-60 %
>60 %


Units:
kW/m
ICELAND
SWEDEN
NORWAY
ATLANTIC
OCEAN
NORTH SEA
DENMARK
BALTIC SEA
REPUBLIC OF IRELAND
UNITED KINGDOM
HOLLAND
GERMANY
POLAND
BELGIUM
FRANCE
CZECH REPUBLIC
No Data
No Data
55
50
41
65
63
67
60
60
57
61
65
22
76
11
68
53
46
20°W
10°W
0°
10°
20°
60°
50°
Wave Power Rose
Season: Annual
0-10 %
10-20 %
20-30 %
30-40 %
40-50 %
50-60 %
>60 %


Wave Power Rose
Season: Annual
0-10 %
10-20 %
20-30 %
30-40 %
40-50 %
50-60 %
>60 %
Units: kW/m
BARENTS SEA
NORWEGIAN SEA
No Data
32
33
50
41
60
61
No Data
1000 m
200 m
SWEDEN
FINLAND
NORWAY
RUSSIA
ESTONIA
0°
10°
20°
30°
40°
70°
60°

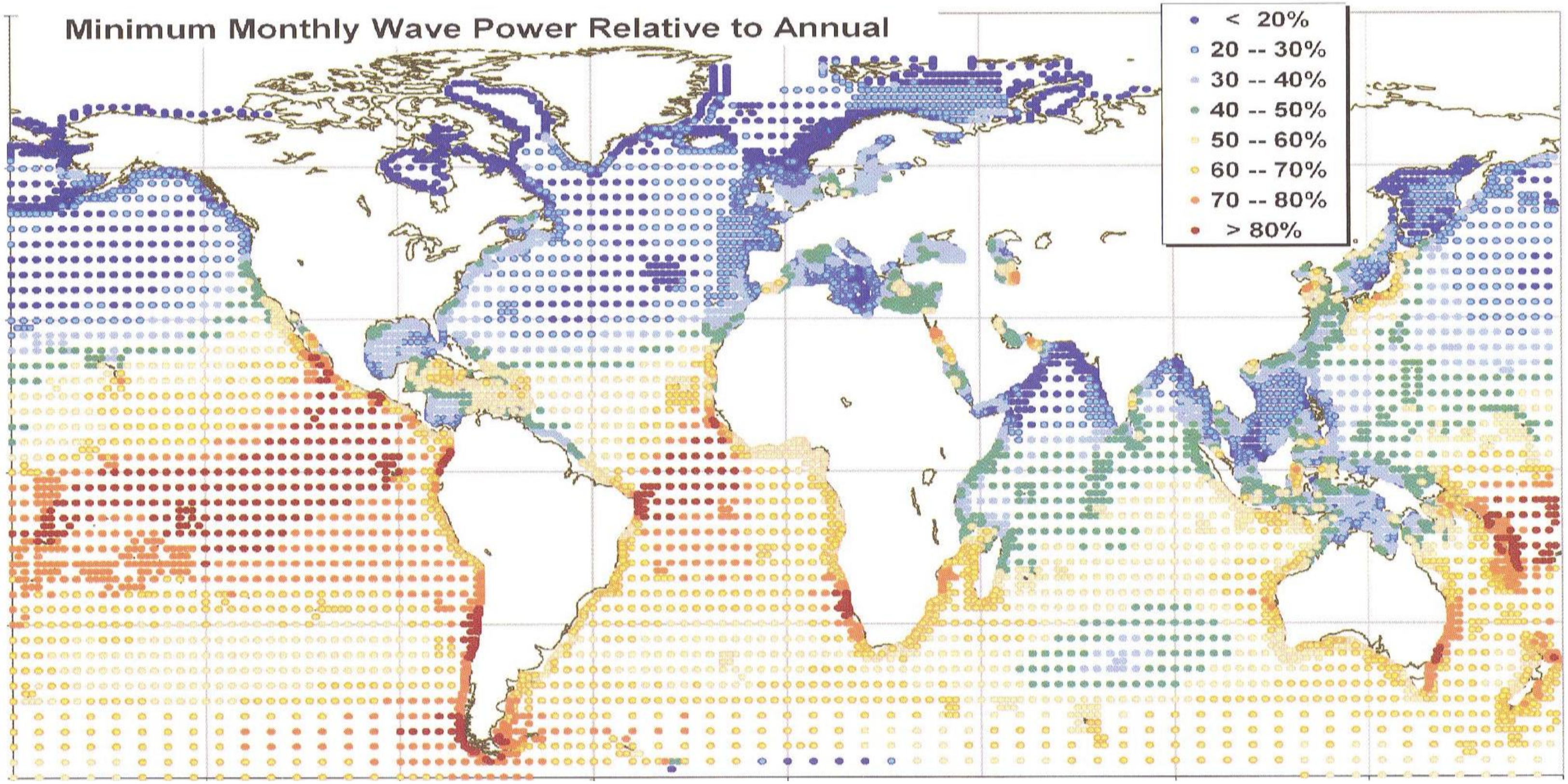

Wave Power Rose
Season: Annual
0-10 %
10-20 %
20-30 %
30-40 %
40-50 %
50-60 %
>60 %
FRANCE
SWITZERLAND
AUSTRIA
HUNGARY
SLOVENIA
ROMANIA
CROATIA
BOSNIA AND HERZEGOVINA
YUGOSLAVIA
ITALY
FYROM
SPAIN
TYRRHENIAN SEA
IONIAN SEA
MEDITERRANEAN SEA
ALGERIA
TUNISIA
MOROCCO
LIBYA
Units: kW/m


Wave Power Rose
Season: Annual
0-10 %
10-20 %
20-30 %
30-40 %
40-50 %
50-60 %
>60 %
Units:
kW/m
HUNGARY
UKRAINE
MOLDOVA
ROMANIA
SEA OF AZOV
SLOVENIA
CROATIA
BOSNIA AND HERZEGOVINA
YUGOSLAVIA
BULGARIA
BLACK SEA
FYROM
ARMENIA
TURKEY
IRAN
IONIAN SEA
SYRIA
LEBANON
IRAQ
MEDITERRANEAN SEA
JORDAN
LIBYA
EGYPT
ISRAEL
20°
30°
40°

Wave Energy Resource

Seasonal Variation

Seasonal variations are much larger in the Northern Hemisphere than in the Southern Hemisphere (an important advantage)

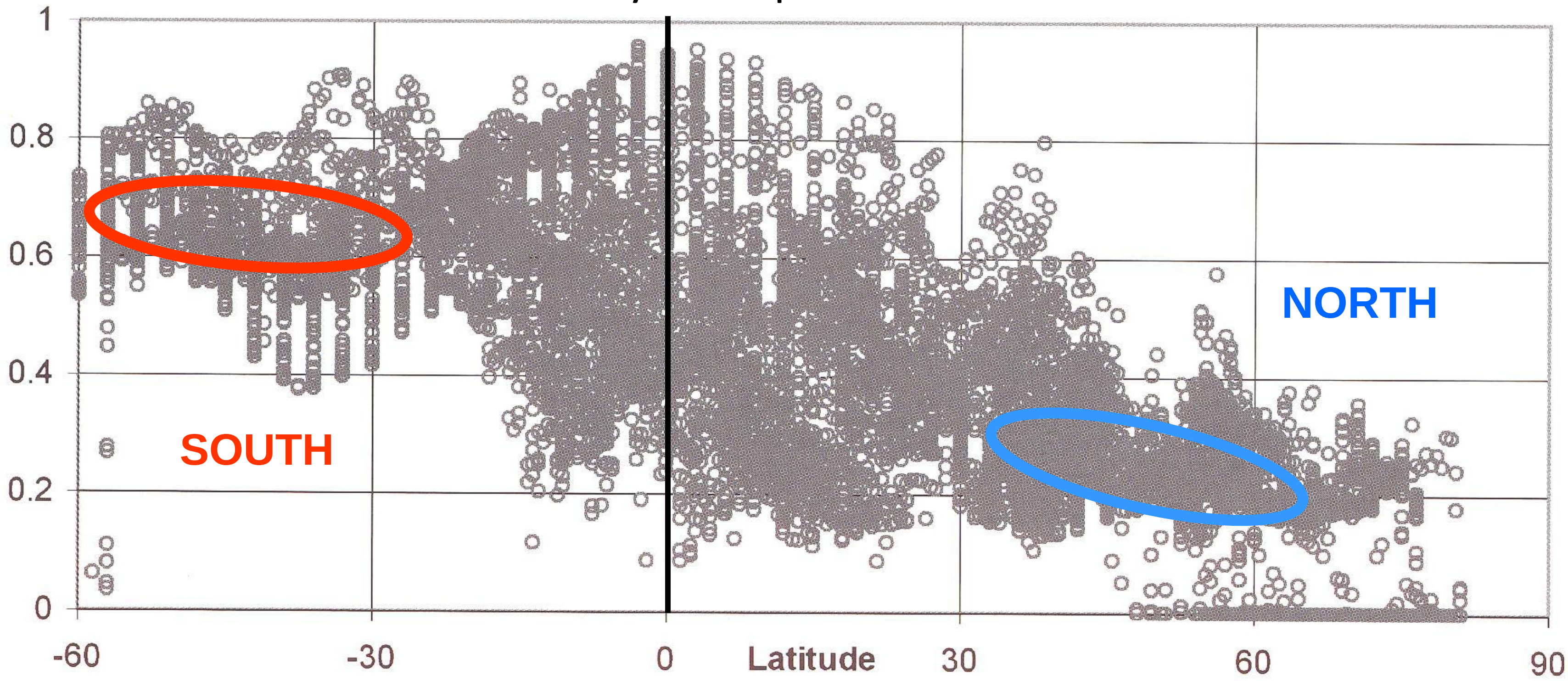


From: Barstow, Mollison & Cruz

Wave Energy Resource

Seasonal Variation

Lowest mean monthly wave power relative to annual mean



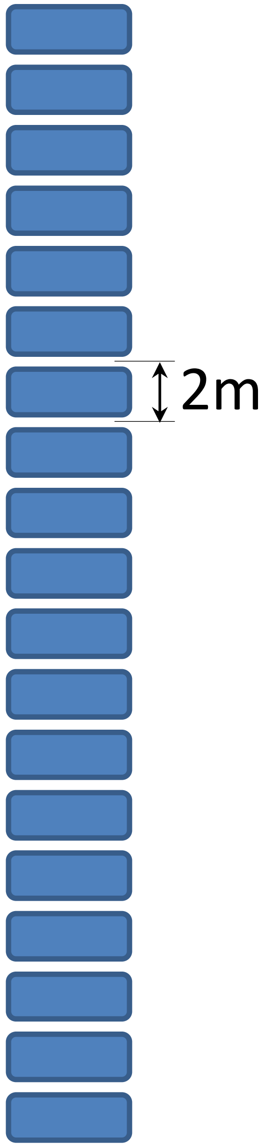
From: Barstow, Mollison & Cruz

How much wave power along the Portuguese coast?

Annual average
40 kW/m

2 x 40 = 80 kW = 108 CV

250 000



THE ENERGY RESOURE

Theoretical Resource - A top level statement of the energy contained in the entire resource

Technical Resource – Part that can be exploited based on existing technologies

Accessible resource – What can be exploited after consideration of external constraints (impractical areas, competing uses, environmentally protected areas, …)

Economic resource – What can be commercially atractive depending on market conditions

ASSESSMENTS WILL DEPEND STRONGLY ON SCENARIOS

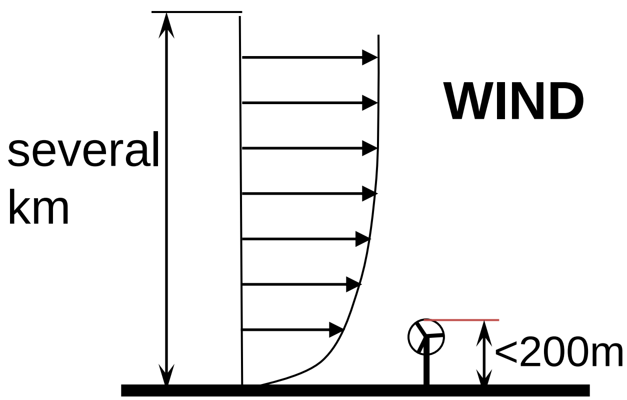
Similarities and contrasts between the wind energy resource and the wave energy resource

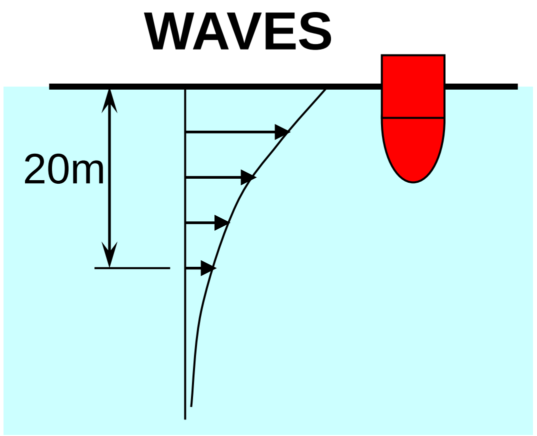
Comparison between time-averages (over tens of minutes to one hour):

Waves result from the integrated action of the **wind** over large ocean areas (thousands of square km) and several hours or days their variability is less than for wind, and they are more predictable

Over time-scales of a few wave periods, the waves are random, like wind turbulence.

Due to the own nature of waves, the absorbable power is highly oscillating and practically discontinuous.





The wind velocity profile extends over several km.

A wind farm explores a tiny sublayer

Most of the wave energy flux is concentrated near the surface

A wave farm can absorb a large part of the wave energy flux.

Typically, the energy flux per unit vertical area for waves near the surface is about 5 times larger than for wind.

Waves are a more concentrated form of energy than wind.

END OF PART 1. WAVE ENERGY RESOURCE